Technicolor on the lattice

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Work done in collaboration with:
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Background:

- The Standard Model (with Higgs) is phenomenologically extremely successful.
- However: The Higgs field is special:
  - *it is the linchpin of the standard model: provides the mechanism for the electroweak symmetry breaking*
  - *it has not been seen*
  - *it is a scalar*

  $\Rightarrow$ *theoretical problems at very high scales: hierarchy problem, vacuum stability, unitarity bound . . .*
- Most BSM models aim to ameliorate these problems by e.g.
  - pairing scalars with fermions (SUSY)
  - introducing a cutoff (extra dimensions)
  - not having scalars at all (Technicolor and many other strongly coupled BSM models)

Chiral symmetry breaking vs. Higgs mechanism

Consider the standard Electroweak symmetry breaking with Higgs and the chiral symmetry breaking (χSB) in QCD:

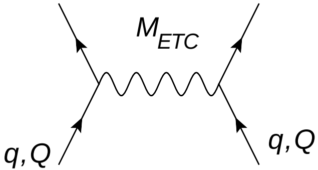
	EWSB	χSB
condensate (Breaks EW):	Higgs vev v	f_π decay constant
Goldstone bosons:	W, Z longitudinal modes	π-mesons
radial excitation:	Higgs particle	scalar meson

Technicolor

- New gauge field (technigauge) + massless fermions (techniquarks) Q.
- Techniquarks have both technicolor and EW charge (exactly like quarks in the SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale: $\Lambda_{\rm TC} \approx \Lambda_{\rm EW}$
- After chiral symmetry breaking:
  - $\Rightarrow$ decay constant $f_{\rm TC} \leftrightarrow$ Higgs expectation value v.
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson $\leftrightarrow$ Higgs
  - $\Rightarrow$ pseudoscalars $\leftrightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons (observable!)
- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Elegant, "proven" mechanism in the Standard Model
- Does *not* explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

Extended technicolor

- In addition to the “pure” technicolor, introduce a new higher-energy interaction coupling Standard Model fermions q (quarks, leptons) and techniquarks (Q): extended technicolor (ETC)
  Several options, e.g. massive gauge boson, $M_{\rm ETC}$:



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- $\frac{g_{\rm ETC}^2}{M_{\rm ETC}^2}\bar{Q}Q\bar{q}q \longrightarrow$ SM fermion mass $m_q \propto \frac{1}{M_{\rm ETC}^2}\langle\bar{Q}Q\rangle_{\rm ETC}$
- $\frac{g_{\rm ETC}^2}{M_{\rm ETC}^2}\bar{q}q\bar{q}q \longrightarrow$ extra FCNC's (harmful!)
- $\frac{g_{\rm ETC}^2}{M_{\rm ETC}^2}\bar{Q}Q\bar{Q}Q \longrightarrow$ explicit χSB in the techniquark sector

$\langle\bar{Q}Q\rangle_{\rm ETC}$: condensate evaluated at the ETC scale
$\langle\bar{Q}Q\rangle_{\rm TC}$: condensate at TC (EW) scale

Extended technicolor

I) $\bar{q}q\bar{q}q$ -term leads to unwanted FCNC's. In order to be compatible with precision electroweak tests, we must have
$\Lambda_{\rm ETC} \approx M_{\rm ETC} \gtrsim 1000 \times \Lambda_{\rm TC} (\Lambda_{\rm TC} \approx \Lambda_{TC})$

II) For EWSB we must have $\langle \bar{Q}Q \rangle_{\rm TC} \propto \Lambda_{\rm TC}^3 \approx \Lambda_{\rm EW}^3$

III) On the other hand, $\langle \bar{Q}Q \rangle_{\rm ETC} \propto m_q \Lambda_{\rm ETC}^2$ (top quark!)

- Using RG evolution

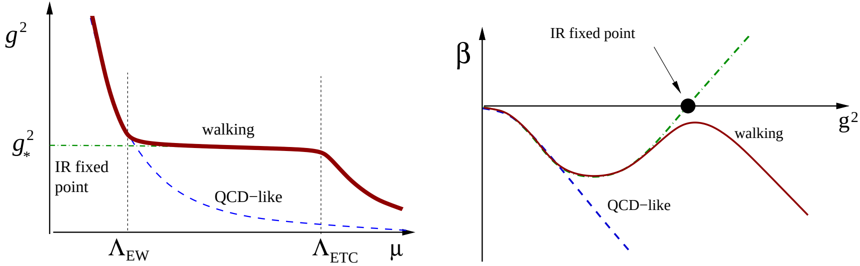
$$\langle \bar{Q}Q \rangle_{\rm ETC} = \langle \bar{Q}Q \rangle_{\rm TC} \exp \left[\int_{\Lambda_{\rm TC}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu \right]$$

  where $\gamma(g^2)$ is the mass anomalous dimension.

- In weakly coupled theory $\gamma \sim 0$, and $\langle \bar{Q}Q \rangle$ is $\sim$ constant.
- *Thus, it is not possible to satisfy the constraints I), II), II) in a QCD-like theory, where the coupling is large only on a narrow energy range above χSB.*

Walking coupling

- If the coupling *walks*, i.e. if $g^2 \approx g_*^2$ (constant) over the range from TC to ETC, then $\langle \bar{Q}Q \rangle_{\rm ETC} \approx \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^{\gamma(g_*^2)} \langle \bar{Q}Q \rangle_{\rm TC}$ (condensate enhancement)
- $\gamma(g_*^2) \sim 1 - 2$ in order to satisfy the conditions I) – III) (depends on the details).



- In a walking theory the β-function $\beta = \mu \frac{dg}{d\mu}$ reaches almost zero near g_*^2.
- If the β-function hits zero there is an IR fixed point, where the system becomes *conformal*.

Perturbative β-function

2-loop universal β-function for SU(N_c) gauge theory with N_f fermions:

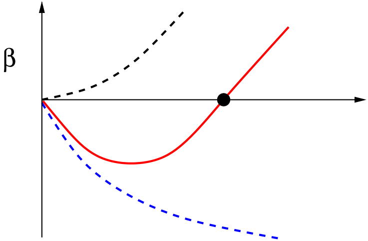
$$\beta(g) = -\mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

where the coefficients are

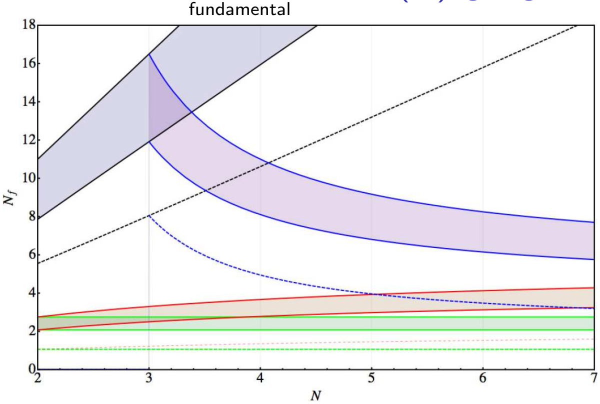
$$\beta_0 = \frac{11}{3} C_r - \frac{4}{3} T_r N_f, \qquad \beta_1 = \frac{34}{3} C_r^2 - \frac{20}{3} C_r T_r N_f - 4 C_r T_r N_f$$

When N_f is varied, generically 3 different behaviours seen:

- confinement and χSB at small N_f
- IR fixed point (conformal window) at medium N_f [Banks,Zaks]
- Asymptotic freedom lost at large N_f



Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

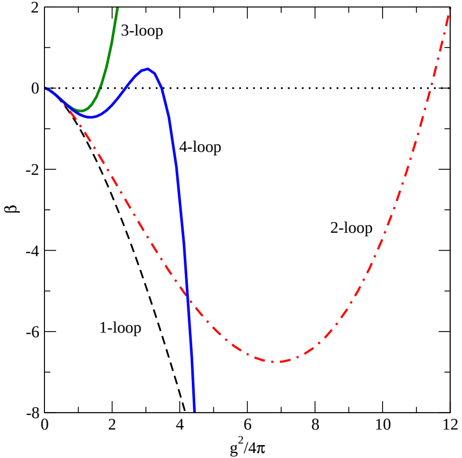
2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich] → lot of recent activity!

Existence of the IRFP essentially non-perturbative

Example: Perturbative β-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions



[4-loop MS: Ritbergen, Vermaseren, Larin]

Preliminary results from lattice: IRFP exists, $g_*^2/4\pi \sim 0.8$ [Karavirta et al]

What do we want?

Take SU(N) gauge theory with N_f fermions in some representation.

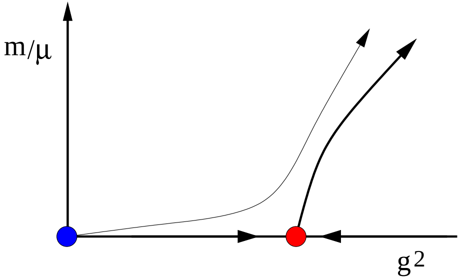
- Measure $\beta(g^2)$-function
- Measure $\gamma(g^2)$
- Classify QCD-like / walking / conformal
- We want to find a theory which
  - is walking or
  - is just within conformal window (easy to deform into walking)
  - has large anomalous exponent γ near FP
  - Compatible with EW precision measurements (S,T,U -parameters) $\to$ small N_f preferred!
- Favourite candidates: SU(2) or SU(3) gauge theory with $N_f = 2$ adjoint or 2-index symmetric representation fermions. [Sannino,Tuominen,Dietrich]

Models studied on the lattice

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: χSB [Appelquist et al; Deuzeman et al; Fodor et al; Jin et al]
  - $N_f = 9$: χSB [Fodor et al]
  - $N_f = 10$: unclear [Yamada et al]
  - $N_f = 12$: conflicting results [Hasenfratz; Fodor et al; Appelquist et al; Deuzeman et al]
  - $N_f = 16$: conformal [Damgaard et al; Heller; Hasenfratz; Fodor et al]
- SU(2) + fundamental rep fermions:
  - $N_f = 2$: χSB [many]
  - $N_f = 3$: conformal(?) [Iwasaki et al]
  - $N_f = 6$: conformal(?) [Del Debbio et al, Karavirta et al (to be published)]
  - $N_f = 8$: conformal [Iwasaki et al]
  - $N_f = 10$: conformal [Karavirta et al (to be published)]
- SU(2) + $N_f = 2$ adjoint rep: (*Minimal walking technicolor*) conformal [Catterall et al; Bursa et al; Hietanen et al]
- SU(3) + $N_f = 2$ 2-index symmetric rep: unclear [de Grand et al; Sinclair and Kogut]

RG flow in conformal case

- Relevant parameters at UV: g^2 and m_Q



- m_Q is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$
- New UV fixed point at stronger coupling? [Kaplan et al; Lombardo et al; Hasenfratz]

Case study: Minimal walking technicolor (MWTC)

- SU(2) with $N_f = 2$ *adjoint* representation techniquarks
- Study on the lattice using (unimproved) Wilson fermions
  [Catterall, Sannino; Del Debbio, Patella, Pica; Hietanen et al, Bursa et al]
- What is studied?
  - Measure the evolution of the coupling directly using the Schrödinger functional method
  - Particle spectrum: do we observe chiral symmetry breaking (QCD) or do all modes become massless as $m_q \to 0$ (no χSB, possibly conformal)
  - Mass anomalous exponent γ (from spectrum or directly using SF methods)
  - Improvement of the lattice action

Evolution of the coupling

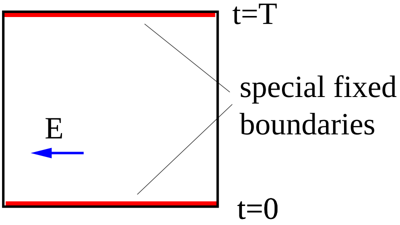
Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised by a twist angle η

At the classical level, we have

$$\frac{dS_{\mathrm{class.}}}{d\eta} = \frac{A}{g^2}$$

where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\begin{aligned}\frac{1}{g^2} &= \frac{1}{A}\frac{dS}{d\eta} \\ &= \text{const.} \times \langle(\text{boundary plaq.})\rangle\end{aligned}$$

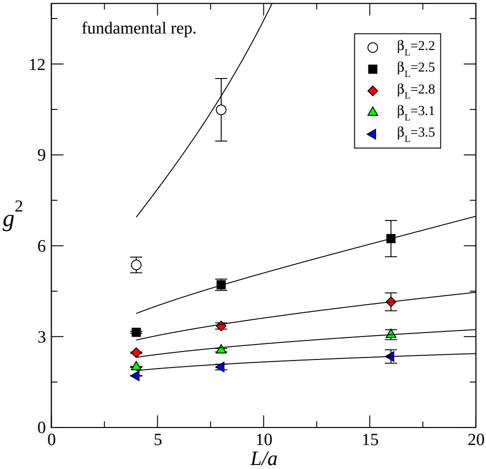


- Evaluates g^2 directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Evolution of the coupling: QCD-like

Test with $N_f = 2$ **fundamental representation**, QCD-like test case:

- L/a grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \to$ small lattice spacing $\to$ small volume
- Continuous line: coupling evaluated from the 2-loop β-function (integration constant fixed to measurement at $L/a = 16$)
- Not a continuum limit, but shows consistency



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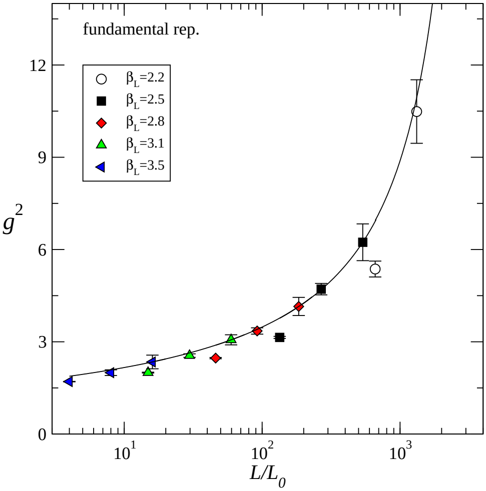
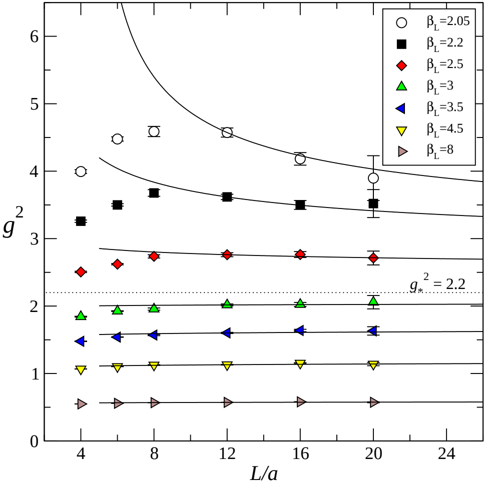


Evolution of the coupling: MWTC

In **adjoint representation**:

- At small $g^2(L)$: increases with L (asymptotic freedom)
- At large $g^2(L)$: *decreases* as L increases
  $\Rightarrow$ β-function positive here!
- Large cutoff effects at small L/a – discard
- As $L/a \to \infty$, apparently $g^2(L) \to g_*^2 \approx 2 \ldots 3$.
  $\Rightarrow$ conformal behaviour!?
- Continuous line: coupling evaluated with fitted β-function ansatz (to be described)

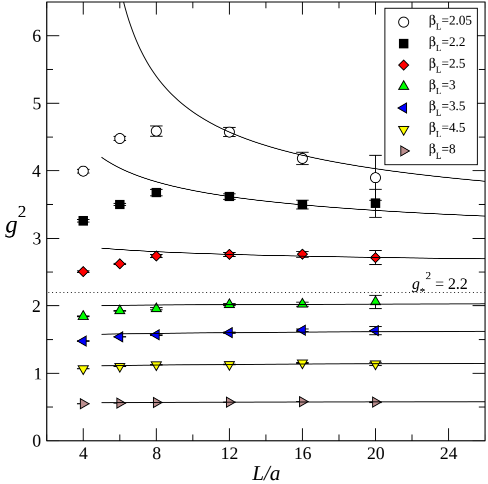


β-function

Assuming that the lattice effects on the large-volume data are small, we can describe the features of the β-function by fitting an ansatz:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

Here b_1, b_2 are perturbative constants and b_3 and δ are fit parameters. (Parametrising the location of the fixed point and the slope of the β-function there).
The ansatz is fitted to the data at $L/a = 12, 16, 20$:



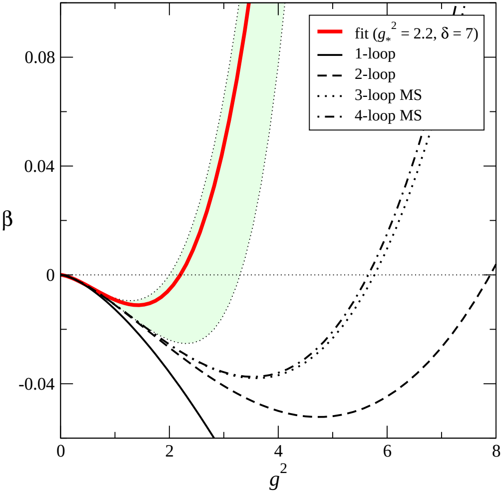
β-function

Fit result:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

FP is at substantially smaller coupling than indicated by 2-loop P.T.

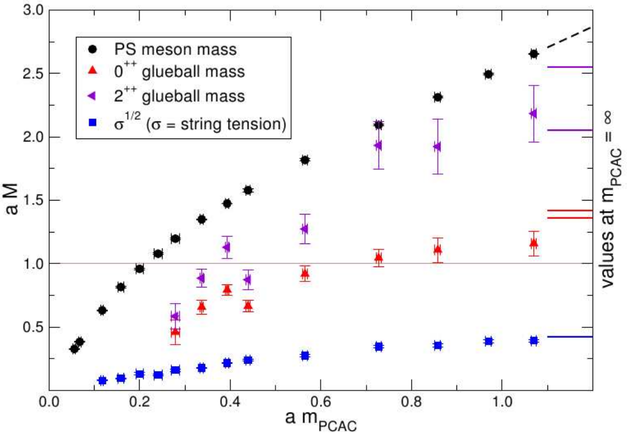
In MS-schema, β-function is known to 4-loop order: [Ritbergen, Vermaseren, Larin]
Not directly comparable to lattice (beyond 2 loops), because of different schema! But quantifies perturbative uncertainty.



Particle spectrum:

- If QCD-like χSB: as $m_Q a \to 0$,
  - $m_\pi \propto m_Q^{1/2}$
  - other states have finite mass.
- If IR fixed conformal point: when $m_Q a \to 0$, all states become massless with the same exponent.
- If walking behaviour: at high energy $\sim$ conformal, at small χSB.

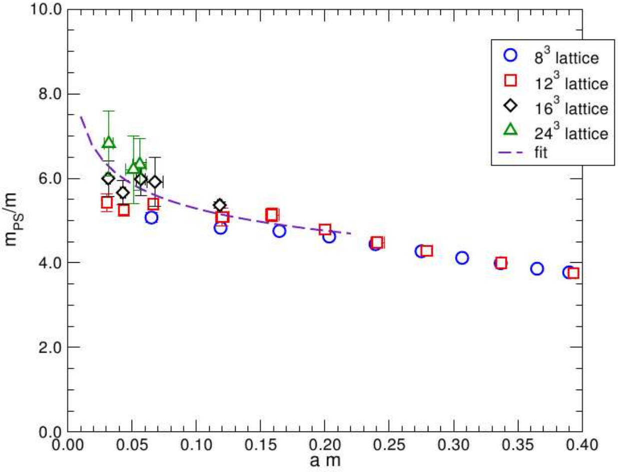
Mass spectrum at $\beta_L = 2.25$



[Del Debbio et al]

Spectrum becomes massless, inverted hierarchy when compared with QCD [Miransky]

Testing scaling of the mass:γ



$$\log m_{\rm PS} = \frac{1}{1+\gamma} \log m + C = 0.85 \log m + C$$

The mass anomalous exponent $\gamma \sim 0.2$

γ can be measured directly using Schrödinger functional scheme, with comparable

What does the results imply?

- Clear hints of a conformal infrared fixed point at $g_*^2 \approx 2 - -3$ in SF schema.
- Exponent $\gamma(g_*^2) \ll 1$. Too small for succesful technicolor.
- However: the results have been obtained independently, but using similar lattice action. Unknown systematics!
- Strong systematic effects have been observed e.g. in simulations of SU(3) with 2 sextet fermions.
- *Must have very careful control of the systematics and the continuum limit* $\rightarrow$ *improved actions*

Conclusions

Lattice technicolor and conformality:

- Lot of work has been done, signs of IRFP found in several theories.
- No clear sign of proper walking found in any (massless) theory
- Anomalous dimensions appear to be small (bad for TC)
- Methods still under development; e.g. there are several methods used to measure the evolution of the coupling. Good for cross-checking, but no "golden" method known
- Need to live at strong coupling: use an action which minimizes lattice effects there. Improvement!
- Cross-checking needed!

Solid results obtainable with present-day resources and methods:

- improved computational methods
- $\sim$ teraflops-year numerical effort

Lattice simulations can exclude models from contention!