HARD THERMAL LOOPS AND THE SPHALERON RATE ON THE LATTICE

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- $T > T_{c,\text{Electroweak}}$: the baryon number of the SM is not a conserved quantum number.
- *Sphaleron rate* Γ
- Topological $\mapsto$ non-perturbative, momentum scale $k \sim g^2 T$
- Almost classical:

$$n_k = \frac{1}{e^{k/T} - 1} \sim \frac{T}{k} \sim \frac{1}{g^2} \gg 1$$
$$\epsilon_k = k \times n(k) \sim T$$

(but not quite...)

Classical system → simulate with classical equations of motion:

i) Select initial configurations from the distribution

$$p \propto e^{-H/T}, \quad H = \frac{1}{2}\int \vec{E}^2 + \vec{B}^2$$

ii) Compute time evolution with the e.o.m

$$D_\mu F^{\mu\nu} = 0 \quad \mapsto \begin{cases} \partial_0 \vec{A} &= -\vec{E} \quad (\text{in } A_0 = 0 \text{ gauge}) \\ \partial_0 \vec{E} &= \vec{D} \times \vec{B} \\ \vec{D} \cdot \vec{E} &= 0 \end{cases}$$

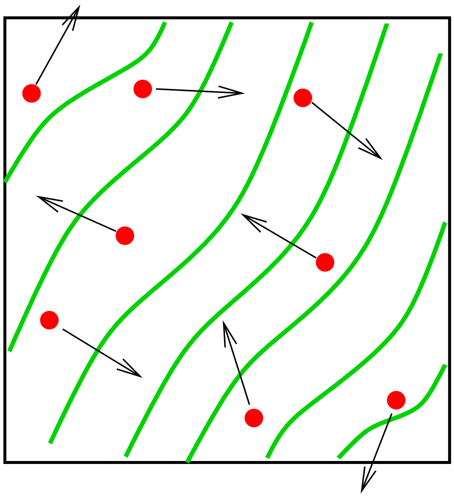
[**SU(2) gauge: Ambjørn and Krasnitz; Tang and Smit; G.D. Moore**]

- UV sector is plain wrong (Rayleigh-Jeans!)

→ the success relies on the decoupling of the hard UV modes ($k \sim T$) from the soft modes ($k \sim g^2T$). This decoupling does occur in the *static* magnetic sector.

- In time-dependent quantities this is *not* the case

Hard Thermal Loop effective theory



Since $T \gg g^2 T$, hard modes $\longrightarrow$ *particles* on the background of soft fields

- adjoint SU(2) charge
- $\vec{v} = \vec{k}/|\vec{k}| = 1$, $v = (1, \vec{v})$
- *Lattice*: Moore, Hu, Müller

- $n(t, \vec{x}, \vec{k})$ = distribution of hard particles. n obeys conserved flow equation (Vlasov):

$$\frac{\mathrm{d}_{\mathrm{conv}} n}{\mathrm{d}t} = 0 \; = \; \partial_0 \delta n + \vec{v} \cdot \vec{D} \delta n + \partial_0 \vec{k} \, \frac{\partial n}{\partial \vec{k}}$$
$$= \; v \cdot D \delta n + g \vec{v} \cdot \vec{E} \, \frac{\partial n_0}{\partial k}$$

where $n = n_0 + \delta n^a$, $n_0 = 1/(e^{-k/T} - 1)$.

- YM equation:

$$D_\mu F^{\mu\nu} = j^\nu_{\mathrm{hard}} = 2 g C_A \int d\vec{k} \; v^\nu \delta n$$

- Factorizing $\delta n(x, \vec{k}) \mapsto -g W(x, \vec{v}) (\partial n_o / \partial k)$ we finally obtain ...

…the Blaizot-Iancu equations:

$$D_\mu F^{\mu\nu} = m_D^2 \int \frac{d\Omega}{4\pi} v^\nu W(x,\vec{v})$$
$$v^\mu D_\mu W(x,\vec{v}) = \vec{v}\cdot\vec{E}$$

Hamiltonian (+ fancy Poisson brackets):

$$H = \tfrac{1}{2}\int d\vec{x}\left[\vec{E}^2 + \vec{B}^2 + m_D^2 \int \frac{d\Omega}{4\pi} W^2\right]$$

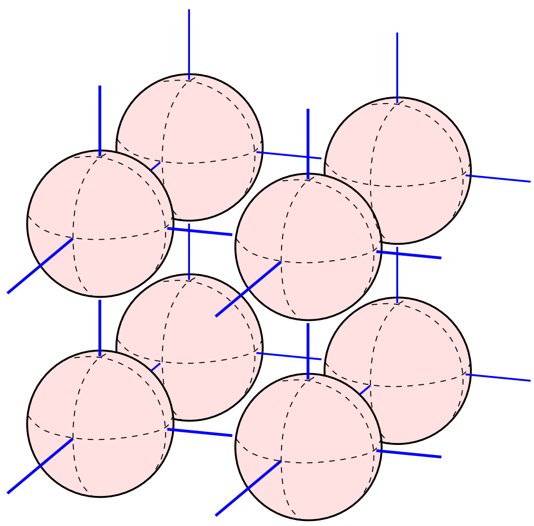
When averaged over initial configurations, static thermodynamics is given by

$$Z = \int dA \exp\left[-\frac{1}{2T}\int d^3x(\vec{B}^2 + (D_i A_0)^2 + m_D^2 A_0^2)\right]$$

Field $W^a(t,\vec{x},\vec{v})$ lives on $\mathbf{R}^3 \times S^2 \longrightarrow$ Discretization:

$$W(x,\vec{v}) = W_{lm}(x) Y_{lm}(\vec{v})$$

Cutoff: if $l \le l_{\max}$, there are $(l_{\max}+1)^2$ components.



Finally, in terms of $W^a_{lm}(t,\vec{x})$, e.o.m is

$$
\begin{aligned}
-\partial_0 \vec{A} &= \vec{E} \\
-\partial_0 \vec{E} + \vec{E} \times \vec{B} &= m_D^2 \, V_m^{i\,*} \, \vec{W}_{1m} \\
\partial_0 W_{lm} &= -C^{LM}_{lm;i} \, D_i W_{LM} + \delta_{l,1} \, V^i_M \, E_i \\
\vec{D} \cdot \vec{E} &= m_D^2 W_{00} \qquad \text{Gauss' law}
\end{aligned}
$$

$$C^{LM}_{lm;i} = \int d\Omega \, Y^*_{lm} \, v^i \, Y_{LM} \qquad V^i_m = \int d\Omega \, Y_{1m} \, v^i$$

Hamiltonian:

$$H = \frac{1}{2} \int d^3x \left[\vec{E}^2 + \vec{B}^2 + m_D^2 \Sigma_{lm} W^2_{lm} \right]$$

On the lattice:

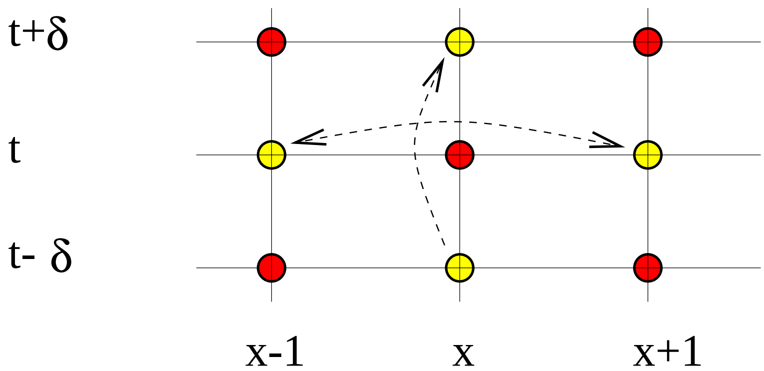
- $3 + 3 + (l_{\max} + 1)^2$ SU(2) matrices per site
- W_{lm} equation is of 1st order $\rightarrow$ *doublers*
- $l_{\max}$ dependence
- a = lattice spacing dependence
- m_D^2 dependence (physics!)
- Still classical $\rightarrow \nexists$ continuum limit

Lattice e.o.m for $W^a_{lm}(t, \vec{x})$:

$$W_{lm}(x, t + \delta_t) - W_{lm}(x, t - \delta_t) =$$
$$\delta_t\{ - C^{LM}_{lm;i}[\mathcal{P}_i W_{LM}(x + i, t) - \mathcal{P}_{-i} W_{LM}(x - i, t)]$$
$$+2\delta_{l,1} v_{mi} E_{\mathrm{ave},i}\} \, .$$

$\mathcal{P}_i W(x+i) = U_i(x) W(x+i) U_i^\dagger(x)$, adjoint parallel transport

Even/odd parity W fields do not talk to each other: $\longrightarrow$ 15 doublers



Doublers are essentially harmless: do not affect the small k gauge field modes.

Conclusions:

- Latticizing Blaizot-Iancu equations in terms of W_{lm} fields is relatively straightforward (parallelizable etc.).
- In SU(2) gauge, the effects of $l_{\max}$ cutoff are surprisingly benign: $l_{\max} = 4$ or 6 seems to be sufficient.
- The results for the *sphaleron rate* in hot SU(2) gauge theory (and, in the SM above symmetry restoration temperature) are remarkably consistent.