Higgs mechanism without a Higgs?
. . . using extra dimensions

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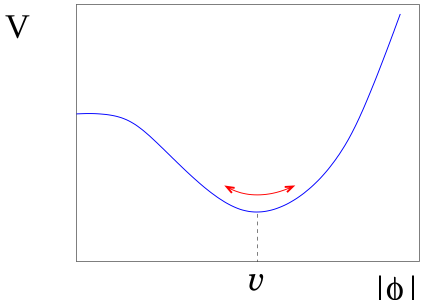
Higgs mechanism

- Spontaneous symmetry breaking: Higgs field expectation value $|\phi| = v \approx 246$ GeV.

$$
\begin{aligned}
|D_\mu \phi|^2 &= |(\partial_\mu + igA_\mu)\phi|^2 \\
&\to \ldots + m_W^2 A_\mu^2
\end{aligned}
$$

where $m_W \sim gv$.



- Fermion masses through Yukawa couplings: $g_Y \phi \bar{\psi}\psi \to m_F \bar{\psi}\psi$
- Only known renormalizable massive vector theory (in 4d)

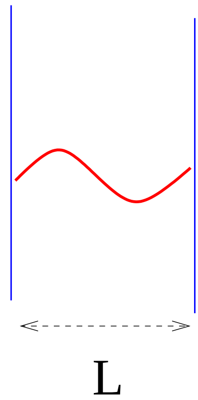
Kaluza-Klein modes in extra dimensions

- Extra dimensions $\rightarrow$ massive modes in lower dimensions.

- For example, if we have one extra periodic dimension of length L, $(d+1)$-dim propagator

$$\Delta_{d+1} = \frac{1}{k_{d+1}^2} \rightarrow \frac{1}{k_d^2 + (2\pi n/L)^2}$$

  looks like a massive propagator in d-dim. if $n \neq 0$.



- Is it possible to arrange things so that *all* masses > 0, and preferably the smallest mass $m_0 \ll m_i$? Yes!

Massive vectors from extra dimensions

[Shaposhnikov and Tinyakov, Phys. Lett. B 515 (2001) 442, hep-th/0102161]

- Let us now consider ***Abelian*** gauge field action in $d+1$-dim.

$$S_{d+1} = \int d^d x \int dz \; \Delta(z) \, \frac{1}{4} F_{AB} F_{AB}$$

where $F_{AB} = \partial_A A_B - \partial_B A_A$ (capital letters denote $d+1$-dim. indices, Greek letters d-dim. indices). I shall use z as the coordinate of the extra dimension.

- Symmetric "weight" function $\Delta(z) > 0$ breaks the Lorentz invariance in $d+1$ -dimensions. Effectively $\boxed{\Delta \sim 1/g^2}$, i.e. the gauge coupling varies as a function of z.

- Expand the gauge field in orthogonal functions:

$$A_B(x,z) = \sum_n A^n_B(x) \; \psi_n(z)$$

We can choose ψ_n to satisfy 2nd order Sturm-Liouville -type equation:

$$-\frac{1}{\Delta}\partial_z[\Delta\ \partial_z\psi_n] = m_n^2\psi_n$$

with normalization

$$\int dz\ \Delta\ \psi_n\psi_m = \delta_{mn} \qquad\qquad \sum_n \psi_n(z)\psi_n(z') = \frac{1}{\Delta}\delta(z-z').$$

- $m^2 \in R$ (hermitean), $m^2 \geq 0$.
- Integrating over z, in gauge $A_z = 0$ action becomes

$$S_{d+1} = \int d^dx \sum_n \left[\frac{1}{4}F^n_{\mu\nu}F^n_{\mu\nu} + \frac{1}{2}m_n^2(A^n_\mu)^2\right]$$

i.e. sum of massive d-dimensional vectors, if $m_n^2 \neq 0$!

- We would like to have:

  1. $m_0^2 > 0$, massive vectors, and
  2. $m_0^2 \ll m_{i>0}^2$, no extra low-energy states.

How to achieve $m_0^2 > 0$:

- $m = 0$ solutions of the Sturm-Liouville eqn

$$-\frac{1}{\Delta}\partial_z[\Delta\,\partial_z\psi] = m^2\psi_0$$

  $\psi =$const. and $\psi = \int^z \Delta^{-1}$.

- If we choose $\Delta(z)$ so that it is not integrable, i.e.

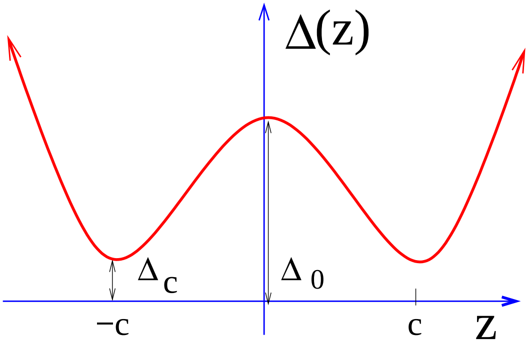
$$\int dz \Delta(z),$$

  diverges, solutions $\psi =$const. and $\psi = \int^z \Delta^{-1}$ are not normalizable. Thus, the lowest mass value $m_0 > 0$.

How about $0 < m_0 \ll m_{(i>0)}$***?***

Achieved with $\Delta(z)$ which has:

- local maximum Δ_0 at $z = 0$
- minimum $\Delta_c \ll \Delta_0$ at $z = \pm c$
- diverges *at least* exponentially as $|z| \to \infty$



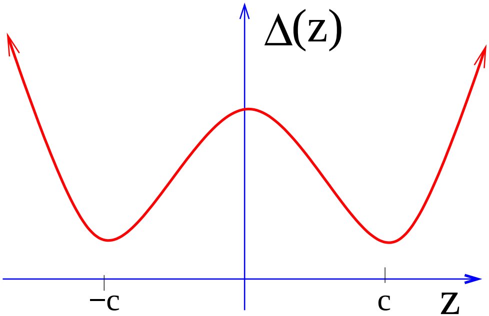
It turns out that $m_0^2 \sim \dfrac{\Delta_c}{\Delta_0} m_i^2$

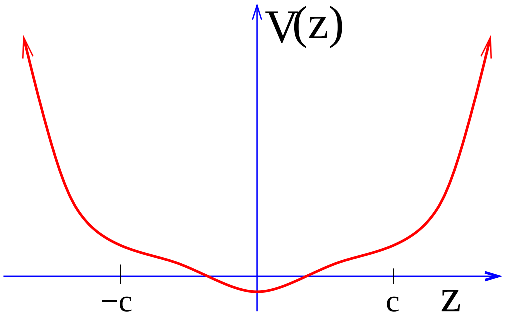
The wave functions and eigenvalues can be analyzed by transforming the equation to Schrödinger form:
defining $\chi = \Delta^{1/2}\psi$,

$$\left[-\frac{d^2}{dz^2} + V(z)\right]\chi = m^2\chi$$

with $V = W^2 - W'$, and $W = -\frac{\Delta'}{2\Delta}$.

V is bounded from below, if Δ grows at least exponentially.





Potential V dips just below 0, and the equation has a solution with $m \gtrsim 0$.

Non-abelian gauge fields

With abelian gauge fields everything is solvable (numerically or analytically, depending on what we choose Δ to be).

However, with non-abelian $A^a_B(x,z) = \sum_n A^{a,n}_B(x)\psi_n(z)$:

A) Action S_{d+1} cannot be decomposed in independent n-modes: there will be interaction terms of type $(\partial_\mu A^n_\nu)A^m_\mu A^k_\nu$, $\quad A^m_\mu\ A^n_\nu\ A^k_\mu\ A^l_\nu$.

$\Rightarrow$ Light modes couple to heavy modes. These ***should*** decouple from the action.

B) The quadratic part of the effective theory for the light modes

$$S_{\text{eff.}} = \int d^dx \Big[\frac{1}{4}F^{a,0}_{\mu\nu}F^{a,0}_{\mu\nu} + \frac{1}{2}m_0^2 A^{a,0}_\mu A^{a,0}_\mu\Big]$$

is (at least superficially) gauge ***non-invariant***. Non-renormalizable $\mapsto$ the heavy modes may not decouple?

C) The original $d+1$ -dim. theory is manifestly gauge invariant. Since local symmetries cannot ***really*** be spontaneously broken, the low-energy theory should be gauge invariant too. This we can achieve by writing the effective theory as a gauge + Higgs theory! Since the Higgs is not a d.o.f. in our system, the effective Higgs mass must be very large ($\sim \infty$). This kind of Higgs sector is strongly coupled and non-perturbative

$\mapsto$ Need to study the problem non-perturbatively: lattice simulations.

- Note: the weight $\Delta(z)$ localizes the light mode around $z = 0$. Thus, the lower-dimensional world lives on the $z = 0$ "brane". Various localizations on planar defects are very well known in condensed matter physics.

Simulations

- Work in $2+1$ dimensions (3d theory renormalizable, easy to manipulate).
- Use U(1) (to compare with analytical calculations) and SU(2).
- We measure the ***static force*** between heavy fundamental charge-anticharge pair: ***Wilson loop*** of size (R,T) at constant z:

$$\begin{aligned} W(R,T;z) &= \left\langle \mathrm{Re}\,\mathrm{Tr}\,\mathcal{P}\exp\left[i\oint A_\mu dx_\mu\right]\right\rangle \\ &= \left\langle \mathrm{Re}\,\mathrm{Tr}\,\mathcal{P}\exp\left[i\sum_n \psi_n(z)\oint A_\mu^n dx_\mu\right]\right\rangle . \end{aligned}$$

The last expression is strictly speaking valid only for abelian theory.

The static potential $V(R,z)$ and the *force* $F(R,z)$ is then obtained from

$$V(R,z) = -\lim_{T\to\infty}\frac{1}{T}\log W \qquad\qquad F(R,z) = \frac{\partial V(R,z)}{\partial R}$$

For abelian theory, the force can be calculated exactly:

$$F(R,z) = \frac{1}{2}\sum_n \psi_n^2(z)e^{-m_n R}$$

Thus, if

– $m_0 = 0$: $F(R,z) \to \frac{1}{2}\psi_0^2(z)$ as $R \to \infty$ ("area law")

– $m_0 > 0$: $F(R,z) \to 0$ ("perimeter law")

We expect this to remain valid (modulo normalization) for non-abelian gauge.

- In the non-abelian case, we can also look at the cubic and quartic self-interactions of the light mode:

$$\alpha_3 \equiv \frac{1}{\psi_0(0)}\int dz\, \Delta(z)\psi_0^3(z)\,, \qquad \alpha_4 \equiv \frac{1}{\psi_0^2(0)}\int dz\, \Delta(z)\psi_0^4(z)\,.$$

For the effective theory to be "close" to a gauge theory, we should have $\alpha_3 \approx \alpha_4 \approx 1$. Remember: if $m_0 = 0$, $\psi_0 =$const. and then $\alpha_3 = \alpha_4 = 1$.

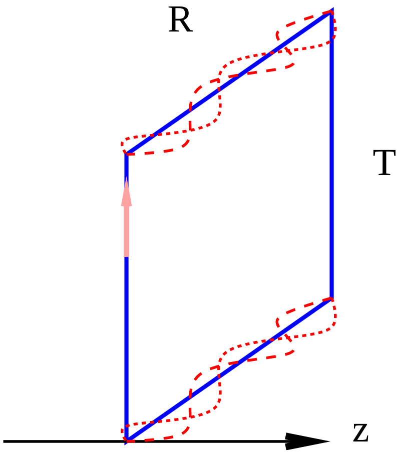
Measurements

We perform simulations in U(1) and SU(2) gauge theory in 2+1 -dim. The force is measured using Wilson loops, where the numerical noise is minimized using

– *link integration* of links to T-direction

– *smearing* of links to R-direction

Mostly we measure at $z = 0$, where the light modes are localized.



3 weight functions:

1. Gaussian: $\Delta(z) = \Delta_0 \exp\left[-\frac{1}{2}m^2 z^2\right]$

2. Sharp: $\Delta(z) = \Delta_0 \exp\left[-M|z| + \frac{1}{2}m^2 z^2\right]$

3. Smooth: $\Delta(z) = \Delta_0 \exp\left[-\frac{1}{2}M^2 z^2 + \frac{1}{4}m^4 z^4\right]$

1. Gaussian weight function

$$\Delta(z) = \Delta_0 \exp\left[-\frac{1}{2}m^2 z^2\right]$$

- Normalizable $\rightarrow m_0 = 0$, $\psi_0(z) =$const.

$\Rightarrow$ The effective theory is massless, $F(R) \rightarrow$const.
Solution in abelian case: $m_n = \sqrt{n}m$.

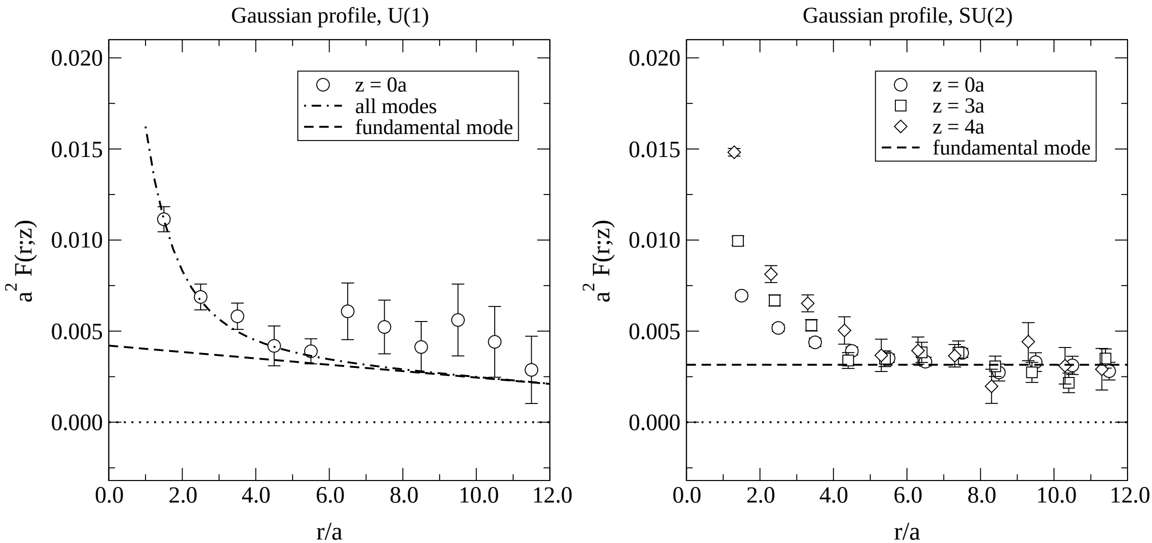
Parameters:

- a = lattice spacing. We want: $m_0 \ll 1/a$, and $a \ll {\Delta_0}$[1]

$$(am)^2 = 0.1, \qquad \frac{4\Delta_0}{a} = 60.0$$

- Lattice sizes: U(1) $48^2 \times 14$, SU(2) $24^2 \times 14$.

[1]In our units $[\Delta] = \mathrm{GeV}^{-1}$; alternatively, if you choose Δ dimensionless (which is equally valid), substitute here $\Delta \mapsto \Delta/g_3^2$.



Force approaches a constant value as $R \to \infty$ in both cases. Low-energy theories are *confining*.

U(1) is not constant, because F must be *antiperiodic* on a finite lattice.

2. Sharp weight function

$$\Delta(z) = \Delta_0 \exp\left[-M|z| + \frac{1}{2}m^2 z^2\right]$$

- Not normalizable $\Rightarrow m_0 > 0$?

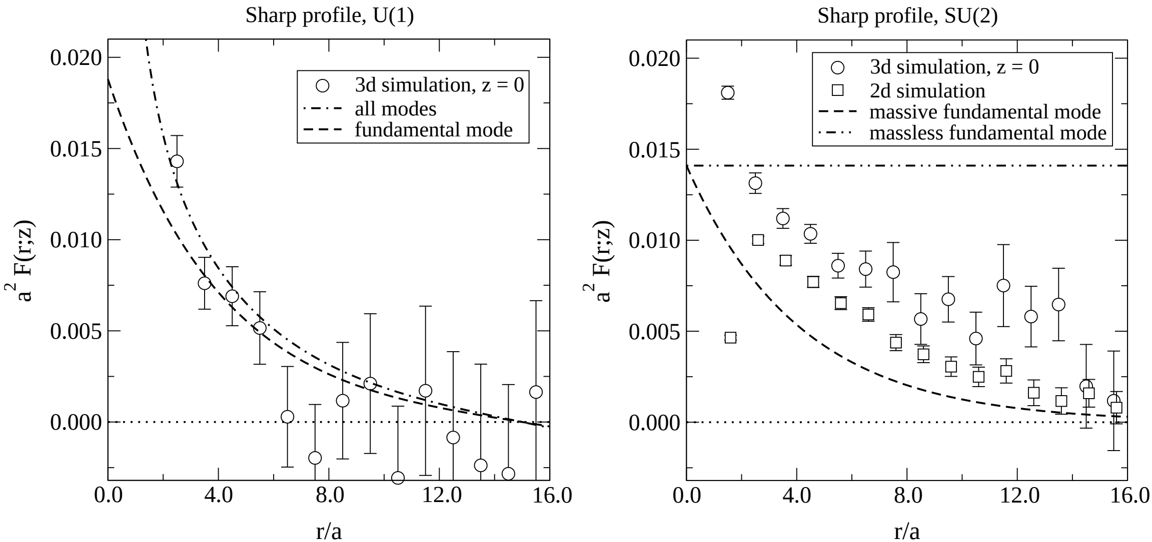
Parameters:

$$am = 0.50, \qquad aM = 0.75, \qquad \frac{4\Delta_0}{a} = 35.0$$

- U(1) numerical solution: $am_0 \approx 0.24$, $am_1 \approx 0.71$.

Thus, $m_0 \ll m_1$ as desired. ($m \ll M \Rightarrow m_0 \ll m_i$)

Cubic and quartic couplings: $\alpha_3 \approx 0.80$, $\alpha_4 \approx 0.75$. These are fairly close to = 1, suggesting that the 2d effective gauge theory should be applicable.



- Clearly, SU(2) force $F \to 0 \implies$ screened (massive) gauge field theory.

- We also compare the results with 2d Higgs model simulation, where the Higgs field expectation value has been set so that the gauge boson mass $= m_0$ ($m_H = \infty$ in this case).

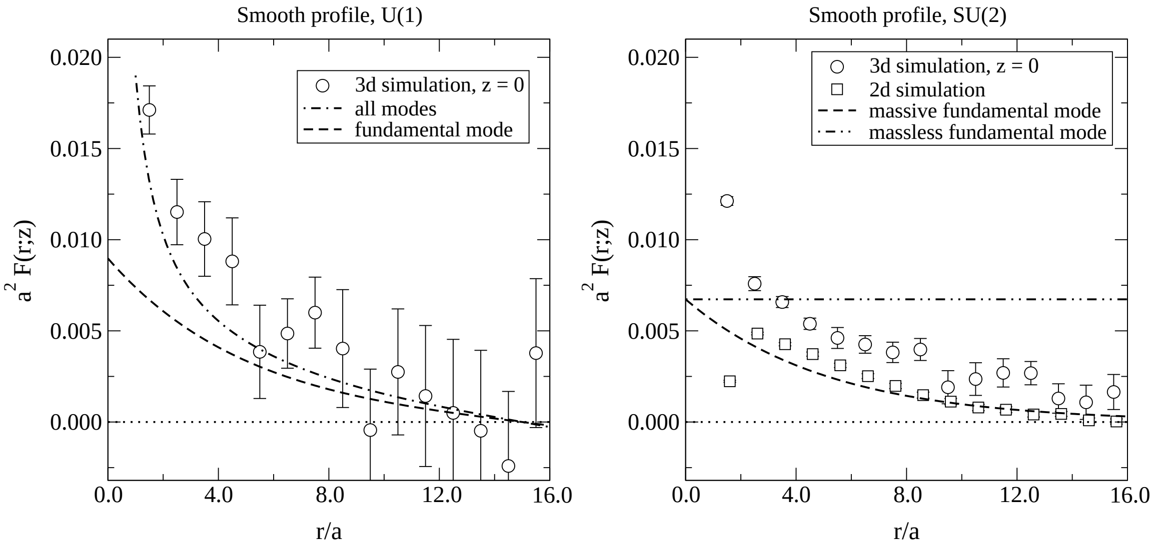
3. Smooth weight function

$$\Delta(z) = \Delta_0 \exp\left[-\frac{1}{2}M^2 z^2 + \frac{1}{4}m^4 z^4\right]$$

Parameters:

$$am = 0.2778, \qquad aM = 0.3889, \qquad \frac{4\Delta_0}{a} = 35.0$$

- U(1) solution: $am_0 \approx 0.19$.
- Cubic and quartic couplings: $\alpha_3 \approx 0.705$, $\alpha_4 \approx 0.695$.



- Again, SU(2) force $F \to 0 \Longrightarrow$ screened gauge field theory.

Conclusions

- Suitable “weight” function $\Delta(z) \sim 1/g^2$ in $(d+1)$ dimensions

  $$\Longrightarrow$$

  effective d-dimensional theory of vector bosons (“W-bosons”), where $0 < m_{\text{vector}} \ll$ higher excitations.

- Light vectors are *localized* on a d-dimensional “brane” at $z = 0$.

- Can be solved analytically for U(1).

- Verified numerically on non-abelian SU(2) theory in $2 + 1$ dimensions.

- SU(2)$\times$U(1) $\rightarrow$ U(1)? Not straightforward.

- Fermion Yukawa masses? Not generated through this mechanism.