Duality and scaling in type II superconductor phase transition

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In collaboration with:

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- Ginzburg-Landau theory (U(1)-gauge + scalar Higgs) in 3 dimensions: model for superconductor phase transition.
  - Coulomb phase $T > T_c$: photon mass $m_\gamma = 0$.
  - Superconducting Higgs phase $T < T_c$: $m_\gamma > 0$, London penetration length $\lambda = 1/m_\gamma$
- Cosmology: GUT, cosmic strings ...
- In type II region, the transition is of 2nd order.
- *Duality* conjecture: transition is in the same universality class as 3d XY model, but with inverse temperature axis.
- Has been difficult to see in experiments and in numerical lattice simulations
- Here: numerical evidence for duality.

Ginzburg-Landau theory

Action of the Ginzburg-Landau (GL) theory, U(1)-gauge + complex scalar:

$$\boxed{S_{GL} = \int d^3x \left[\frac{1}{4}F_{ij}^2 + |D_i\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4\right]}$$

Here ϕ is complex scalar field, $F_{ij} = \partial_i A_j - \partial_j A_i$, and $D_i = \partial_i + ieA_i$. The parameters m, e^2 and λ have dimension GeV.

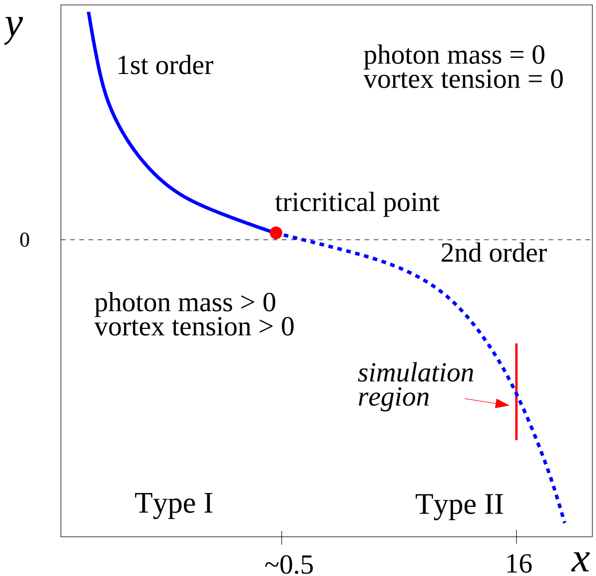
Theory is defined by the dimensionless couplings

$$\boxed{x \equiv \frac{\lambda}{e^2}, \qquad y \equiv \frac{m^2}{e^4}}$$

m^2 is the temperature-like parameter:

- $m^2 < m_c^2$: Superconducting “broken” phase, ($\langle|\phi|\rangle > 0$)
- $m^2 > m_c^2$: Coulombic “symmetric” phase ($\langle|\phi|\rangle = 0$, suitably defined and renormalized).

Phase diagram



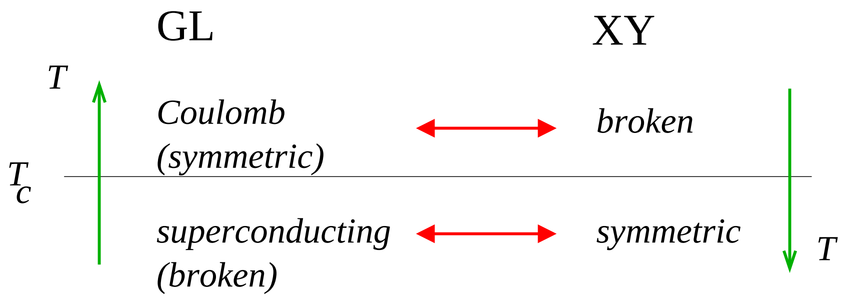
- $x \equiv \lambda/e^2 \lesssim 0.5$: 1st order (type I)

  $x \gtrsim 0.5$: 2nd order (type II)
- No *local* order parameter (like $\langle|\phi|\rangle$ in non-gauged scalar theory).
- Some *non-local* order parameters:
  - photon mass m_γ ($\equiv 1/\lambda$, the inverse London penetration length)
  - Abrikosov-Nielsen-Olesen vortex tension T ($\equiv H_{c,1}$, critical magnetic field)

Duality

It has been argued that the transition in type II region is in the same universality class as 3-dim XY model (or complex scalar theory with O(2) symmetry), but with *inverted* temperature.

Peskin 78; Banks, Myerson, Kogut 77; Dasgupta, Halperin 81; Kleinert 82; Kovner, Rosenstein, Eliezer 91; Kiometzis, Kleinert, Schakel 95, Herbut, Tesanovic 96...



Massless degrees of freedom: *photon* (GL, symmetric phase) $\Leftrightarrow$ *Goldstone mode* (XY, broken phase)

Critical exponents of the GL theory $\equiv$ critical exponents of the XY model.

Duality expected to be valid at very close proximity to the critical point.

Physical observables

Ginzburg-Landau theory		XY model
photon: $1/m_\gamma = \lambda$	$\leftrightarrow$	current-current correlation length
Abrikosov vortex tension $T = H_{c,1}$	$\leftrightarrow$	scalar mass $m_{\rm XY}$
~~*charged scalar mass?*~~	$\leftrightarrow$	~~*vortex tension?*~~
charge-anticharge potential	$\leftrightarrow$	vortex-antivortex potential

Duality maps fundamental d.o.f:s of one theory to non-local topological objects in the other.

In this talk I shall concentrate on

a) Vortex tension $T \leftrightarrow m_{\rm XY}$ scalar mass.

b) Photon mass $m_\gamma \leftrightarrow$ current correlator. (Im$[\phi^* \partial_i \phi]_{\rm XY}$).

As the transition is approached from the broken phase, duality implies that

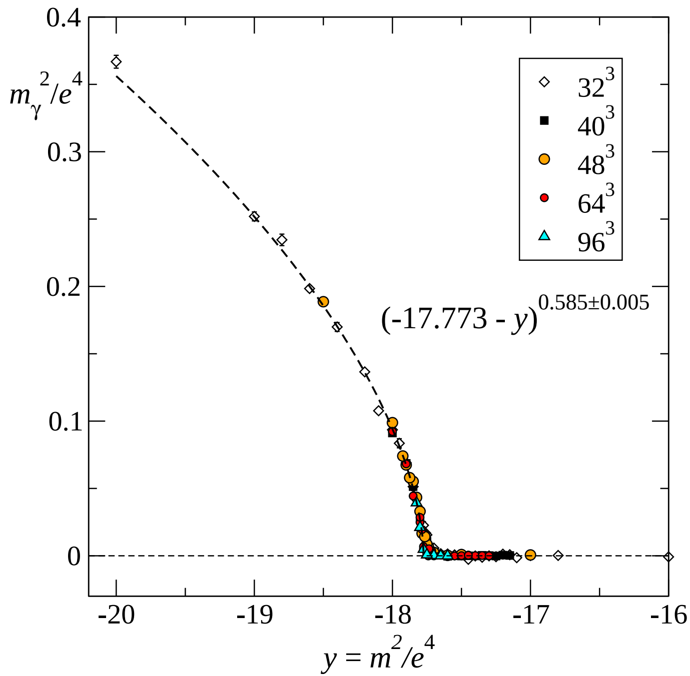
$$\boxed{T \sim |\tau|^{\nu_{\rm XY}}, \qquad m_\gamma \sim |\tau|^{\nu_{\rm XY}}} \qquad\qquad \nu_{\rm XY} \approx 0.6723$$

where $\tau = m^2 - m_c^2$ is the "temperature" parameter, and $\nu_{\rm XY}$ is the well-known XY model critical exponent.

This has *not* been observed in direct lattice MC simulations:

$m_\gamma \sim |\tau|^{0.3} \sim |\tau|^{\nu_{\rm XY}/2}$

[Kajantie et al 01] $\Longrightarrow$



High-T_c superconductor ($YBa_2Cu_3O_{7-\delta}$) experiments have produced

$m_\gamma \sim |\tau|^{0.3}$ [Kamal et al 98]

$m_\gamma \sim |\tau|^{0.4...0.5}$ [Paget et al 99]

What goes wrong?

- duality not valid?
- too far from the transition?
- unforeseen systematics?

On the other hand, in the London limit ($\lambda \to \infty$), where the duality is on a firmer footing, lattice simulations using indirect methods suggest $m_\gamma \sim |\tau|^{\nu_{\rm XY}}$

[Olsson, Teitel 98; Hove, Sudbø 00]

Frozen superconductor

In order to elucidate the issue, we performed lattice simulations with a particular limit of the GL theory, frozen superconductor. This model is *exactly* dual to the XY model in the Villain form.

[Peskin 78; Thomas, Stone 78; Banks, Myerson, Kogut 77]

1) Discretize GL on the lattice (using non-compact gauge)

2) Take limits $\lambda \to \infty$, "hopping parameter" $\kappa \to \infty$

$\Rightarrow$ Integer-valued gauge theory, *frozen superconductor:*

$$\mathcal{Z}_{\mathrm{FZS}}(\beta) = \sum_{\{I_{\vec{x},i}\}} \exp\left(-\frac{\beta}{2} \sum_{\vec{x},i>j} F^2_{\vec{x},ij}\right) ,$$

where $F_{\vec{x},ij} = I_{\vec{x},i} + I_{\vec{x}+i,j} - I_{\vec{x}+j,i} - I_{\vec{x},j}$ and $I_{\vec{x},i} \in Z$.

XY model in the Villain form:

$$\mathcal{Z}_{\mathrm{XY}}(\kappa) = \int D\theta \exp\left(-\kappa \sum_{\vec{x},i} s(\theta_{\vec{x}+i} - \theta_{\vec{x}})\right) ,$$

where $s(x) = -\ln \sum_{k=-\infty}^{\infty} \exp\left(-\frac{1}{2}(x - 2\pi k)^2\right) .$

Exact duality:

It is straightforward to show that

$$\boxed{\mathcal{Z}_{\rm FZS}(\beta) = \mathcal{Z}_{\rm XY}(\frac{1}{\beta})}$$

Even more usefully, we can couple FZS to an external (integer-valued) magnetic field $H_{\vec{x},i}$ in the usual way, using coupling

$$\sum_{\vec{x},i} \vec{B}_{\vec{x}} \cdot \vec{H}_{\vec{x}}, \qquad B_i \equiv \frac{1}{2}\epsilon_{ijk}F_{jk}$$

Duality relation now becomes

$$\mathcal{Z}_{\rm FZS}(\beta, \vec{H}_{\vec{x}}) = \mathcal{Z}_{\rm XY}(\frac{1}{\beta}, \vec{H}_{\vec{x}})$$

where $\vec{H}_{\vec{x}}$ couples to the XY model Noether current ($j_i(\vec{x}) \sim (\theta_{\vec{x}+i} - \theta_{\vec{x}})$, corresponding to Im $\phi^*\partial_i\phi$ in continuum).

This enables us to relate physical observables of the two theories. For example, photon correlation function

$$\langle B_i(x)B_j(y)\rangle = \frac{\delta^2 \ln \mathcal{Z}_{\rm FZS}(\beta,\vec{H})}{\delta H_i(x)\delta H_j(y)} = \frac{\delta^2 \ln \mathcal{Z}_{\rm XY}(\beta,\vec{H})}{\delta H_i(x)\delta H_j(y)} = \langle j_i(x)j_j(y)\rangle$$

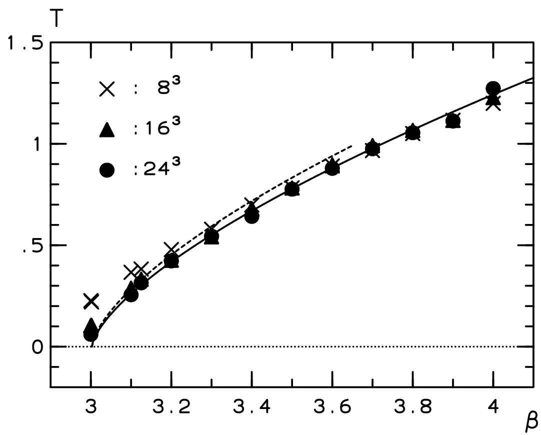
Likewise, we can show that Abrikosov vortex tension = mass of the scalar

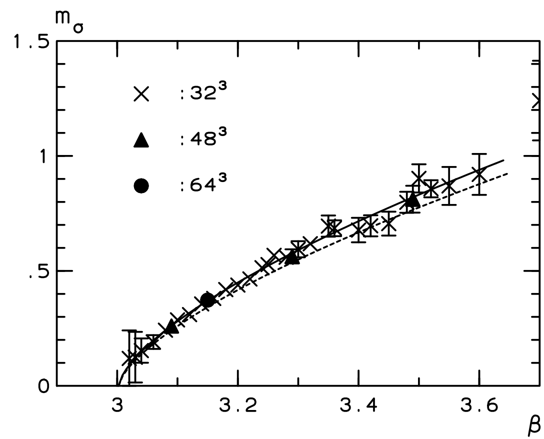
Results

Indeed, Abrikosov vortex tension T in the frozen superconductor and the scalar mass are exactly dual to each other:

FZS vortex tension, $\beta > \beta_c$
(superconducting phase)

XY scalar mass, $\kappa < \kappa_c = 1/\beta_c$
(symmetric phase)

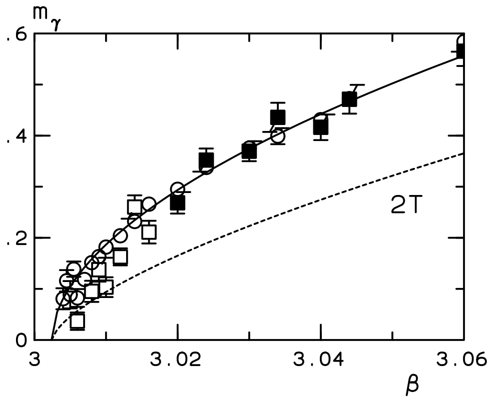




Measured critical exponent $\nu \approx 0.66$

[Neuhaus, Rajantie, Rummukainen 02]

However, the photon mass measurement in FZS gives again something different:



Measured critical exponent: $\nu_\gamma \approx 0.5 \neq \nu_{\mathrm{XY}}$.

This cannot be the final value, because photon can decay into 2 vortices $\nu_\gamma \longrightarrow \nu_{\mathrm{XY}}$.

Culprit for difficulties in photon mass measurements: large anomalous exponent $\eta_A = 1$ in the photon propagator

Constructing dual theory for the Ginzburg-Landau model

[Kajantie, Laine, Neuhaus, Rajantie, Rummukainen 04]

- Ginzburg-Landau $\leftrightarrow$ O(2) symmetric scalar theory (“S”) duality
- In order to relate physics, we must consider the coupling to an external source (magnetic) field H_i.
- The duality conjecture now states that $\boxed{\mathcal{Z}_{\rm GL}(H_i) = \mathcal{Z}_{\rm S}(H_i)}$ near the phase transition point.

GL theory: the standard coupling to the magnetic field is

$$\mathcal{Z}_{\rm GL}(H_i) = \int \mathcal{D}A\, \mathcal{D}\phi^*\, \mathcal{D}\phi\, \exp\left[-S_{\rm GL}\right]$$

$$S_{\rm GL} = \int d^3x \left[\frac{1}{4}F_{ij}^2 + |D_i\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4 + B_i\, H_i\right]$$

where $B_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$. Because $\partial_i B_i = 0$, action is invariant under transformations

$$H_i(x) \mapsto H_i(x) - i\partial_i\alpha(x)$$

where $\alpha(x)$ is an arbitrary real function. This generates a local U(1) symmetry.

Scalar theory: we fix the coupling to H_i by requiring that the action respects the above U(1) symmetry. In the scalar theory, it is natural to assume that this couples to the phase of the complex scalar; i.e. we assume that the dual field transforms as

$$\tilde{\phi} \mapsto e^{i\tilde{e}\alpha}\tilde{\phi}, \quad \tilde{\phi}^* \mapsto e^{-i\tilde{e}\alpha}\tilde{\phi}^*,$$

where $\tilde{e} = 2\pi/e$ (quantization, boundary conditions). Now we can fix the dual scalar theory as

$$S_{\mathrm{S}} = \int d^3x \Big\{ [(\partial_i - \tilde{e}H_i)\tilde{\phi}^*][(\partial_i + \tilde{e}H_i)\tilde{\phi}] + \tilde{m}^2\tilde{\phi}^*\tilde{\phi} + \tilde{\lambda}(\tilde{\phi}^*\tilde{\phi})^2 + \frac{1}{4}\tilde{z}\tilde{F}_{ij}^2 + ... \Big\}$$

$$\mathcal{Z}_{\mathrm{S}}(H_i) = \int \mathcal{D}\tilde{\phi}\mathcal{D}\tilde{\phi}^* \exp -S_{\mathrm{S}}$$

where $\tilde{F}_{ij} \equiv \partial_i H_j - \partial_i H_j$.

- H_i couples to the scalar current $j_i =$ Im$[\,\tilde{\phi}^*\partial_i\tilde{\phi}]$.
- Relation between GL couplings (m^2, λ) and scalar couplings $\tilde{m}^2$, $\tilde{\lambda}$ are not fixed.
- We can still compare critical properties, which are not sensitive to parameter values

- Constant GL magnetic field $H_i \mapsto$ chemical potential in the scalar theory (in 2+1 dim. language) [Son 02]

The duality allows us to give the GL critical quantities in terms of scalar (XY) critical exponents:

- Magnetic permeability (Coulomb phase) $\chi \equiv \partial B/\partial H \sim |\tau|^{\nu_{\mathrm{XY}}}$
- Anomalous exponent $\eta = 1$
- At the critical point, $\frac{\partial}{\partial \tau} pC^{-1}(p) \sim p^{-1/\nu_{\mathrm{XY}}}$
- Vortex tension $T \sim |\tau|^{\nu_{\mathrm{XY}}}$
- Photon mass in superconducting phase (pole of the propagator) $m_\gamma \sim |\tau|^{\nu_{\mathrm{XY}}}$
- …

Photon propagator

GL theory: photon correlation function

$$C_{ij}(x-y) = \langle B_i(x) B_j(y) \rangle$$

Fourier transform of C has the structure

$$C_{ij}(p) = \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) \frac{p^2}{p^2 + \Sigma}$$

In the superconducting phase ($m^2 < m_c^2$), $\Sigma = m_\gamma^2$, and at the critical point

$$\Sigma(p) = A|p|^{2-\eta} + \ldots$$

$\eta =$ anomalous exponent.

Scalar theory: using the duality, we obtain the photon correlation function fully in terms of the scalar theory quantities:

$$C_{ij}(p) = -\tilde{z}p^2 + 2\tilde{e}^2 \left\langle \tilde{\phi}^* \tilde{\phi} \right\rangle - \tilde{e}^2 \frac{1}{V} \langle j_i(p) j_j(-p) \rangle$$

Magnetic permeability and anomalous exponent

Consider magnetic permeability

$$\chi \equiv \frac{\partial B}{\partial H} = \frac{1}{V}\frac{\partial^2}{\partial H^2}\ln \mathcal{Z}_{\rm GL} = \lim_{p\to 0} C(p)$$

in the Coulomb (symmetric) phase of the GL theory.

In the scalar theory we are in the broken phase. Thus, as $p \to 0$, the current correlator is dominated by the Goldstone mode:

$$\langle j_i(p) j_j(-p)\rangle \sim \frac{p_i p_j}{p^2}\tilde{e}^2 \langle \tilde{\phi}^* \tilde{\phi}\rangle$$

Thus, we obtain

$$\chi \propto \langle \tilde{\phi}^* \tilde{\phi}\rangle \sim \xi \sim |\tau|^{\nu_{\rm XY}}$$

where ξ is the scalar model correlation length. Note: $\langle \phi^* \tilde{\phi}\rangle \propto$ helicity modulus of the scalar theory.

[Son 02]

Anomalous exponent can now be obtained by simple finite volume analysis:

- Consider $p \sim 1/L$, L system size
- If $L \gg \xi$, we are essentially at infinite volume and

  $C(p) \approx \chi \sim \xi$
- If $L \ll \xi$, we are effectively at critical point and

  $C(p) \approx p^2/(A|p|^{2-\eta}) \sim |p|^{-\eta} \sim L^\eta$
- Continuity at $L \sim \xi$:

  $\xi \sim \xi^\eta$

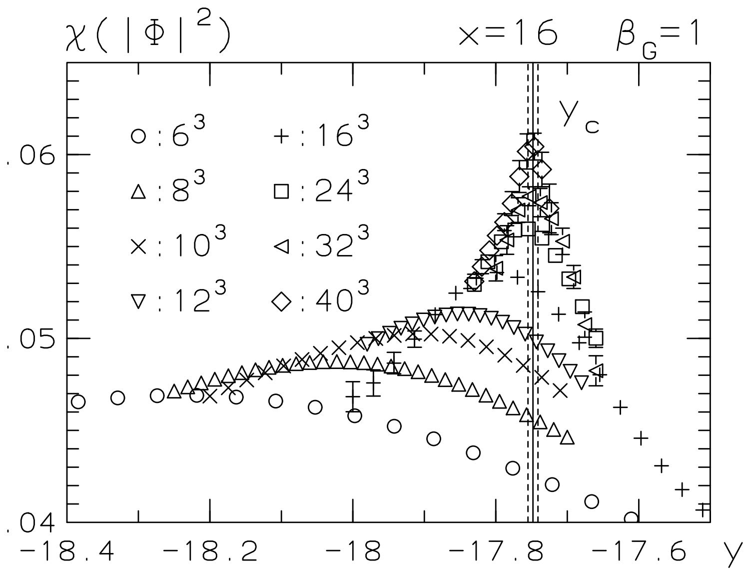
$\Rightarrow$ $\boxed{\eta = 1}$

[Son 02; Appelquist, Pisarski 81; Bergerhoff,Freire, Litim, Lola, Wetterich 95; Herbut, Tesanovic 96]

Simulations of the GL theory

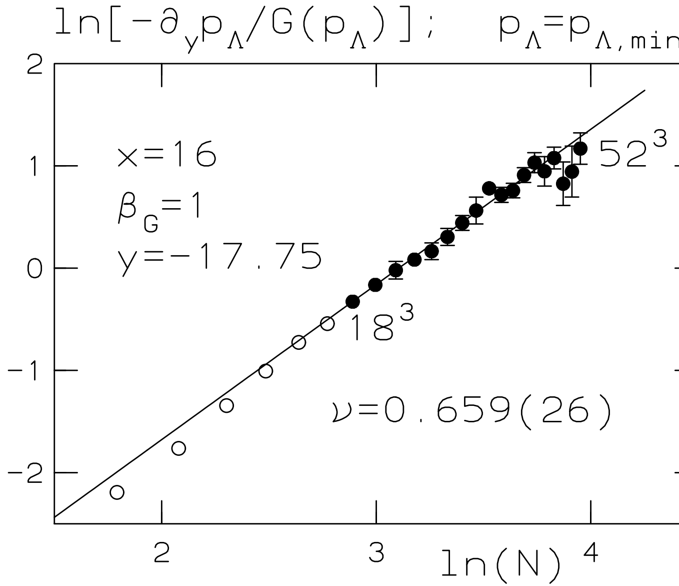
- Non-compact gauge action
- We use lattice spacing $a = 1/e$ ($\beta = 1$) only – not a problem for critical phenomena
- (theory superrenormalizable: continuum limit would not be a problem)
- we use $x \equiv \lambda/e = 16$, well within type II

We determine the critical point from the maximum location of the susceptibility of $\phi^*\phi$:



$y_c \equiv m_c^2/e^4 \approx -17.75$

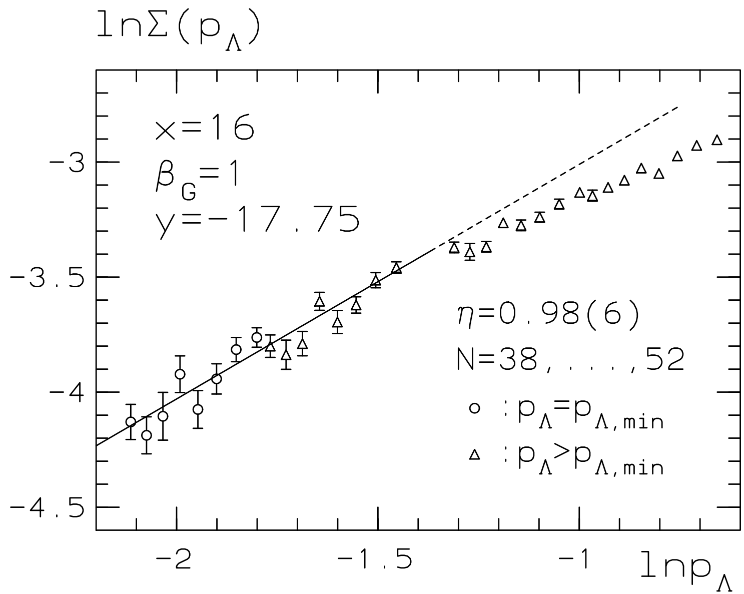
$$\frac{\partial}{\partial \tau} p C^{-1}(p) \sim p^{-1/\nu} \sim L^{1/\nu}$$ at the critical point:



Fitted exponent $\nu = 0.659 \pm 0.026$ perfectly agrees with the XY model exponent $\nu_{\mathrm{XY}} \approx 0.67155$

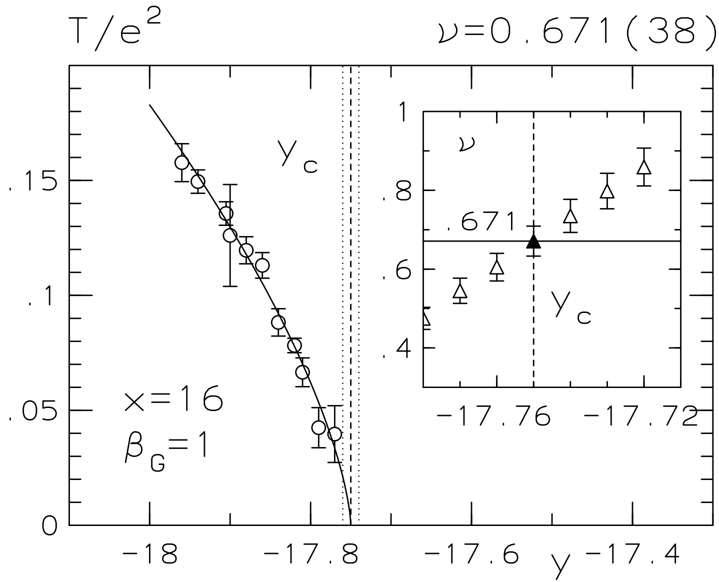
Anomalous dimension at the critical point:

$$C(p) \approx \frac{p^2}{p^2 + \Sigma}, \quad \Sigma \approx A|p|^{2-\eta}$$



Measured $\eta = 0.98 \pm 0.06$

Abrikosov-Nielsen-Olesen vortex tension in the superconducting phase:



critical exponent $\nu = 0.671 \pm 0.038$

Why photon mass is difficult?

The inverse propagator is

$$p^2 C^{-1}(p) = p^2 + \Sigma = p^2 + m_\Sigma^2 + A|p|^{2-\eta} + \dots$$

where $\eta = 1$, and $m_\Sigma^2 \sim |\tau|^{\nu_{\mathrm{XY}}}$.

In order to see the “right” critical behaviour, the propagator must be dominated by the anomalous term:

$$p^2 \ll A|p|^{2-\eta} \approx 0.13 e^2 |p| \qquad (\text{in our case})$$

In this case the photon mass can be obtained from the pole,

$$m_{\mathrm{pole}} \sim |p| \sim m_\Sigma^2 \sim |\tau|^{\nu_{\mathrm{XY}}}$$

If this is *not* the case and the propagator is dominated by the p^2-term,

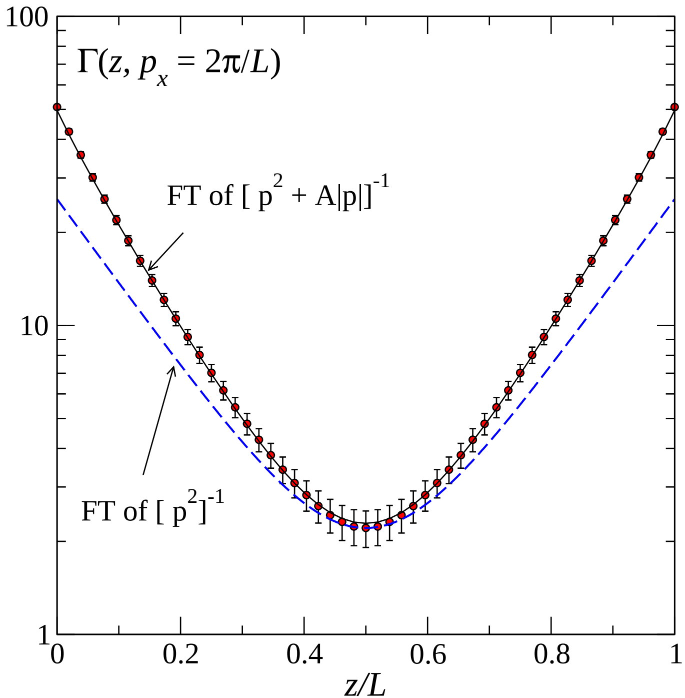
$$m_{\mathrm{pole}} \sim |p| \sim m_\Sigma \sim |\tau|^{\nu_{\mathrm{XY}}/2} \sim |\tau|^{0.33}$$

which is just what has been observed!

The correct critical behaviour requires:

- large volume $L \sim 2\pi/p \gg 50$ lattice units in our case
- small pole mass $m_\gamma \sim p$, i.e. close enough to the critical point.

Propagator at the critical point:



Conclusions

- Simulation results are a perfect match with XY-duality conjecture
- Magnetic field $\leftrightarrow$ chemical potential
- Anomalous exponent of the gauge field propagator $\eta = 1$ explains the difficulties seen in lattice simulations and experiments(?)
- Experiments still too far away from the critical point?