THE ELECTROWEAK PHASE TRANSITION: PRECISION RESULTS

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- Symmetry restoration in the standard model at high T
  [Kirzhnitz, Linde, PLB 72 (1972)].
- Cosmology: $T \sim 100 - 200\,\mathrm{GeV}$; $t \sim 10^{-11}\,\mathrm{s}$.
- EW interactions violate the baryon number B.
  Electroweak baryogenesis?
  – very sensitive to the qualitative and *quantitative* details of the transition:
  $\longrightarrow$
  accurate and reliable results about the thermodynamics of the transition are needed!

- What do we know about the transition:
  - For small m_H, the transition is of 1st order.
  - For $m_H \gtrsim 75\,\mathrm{GeV}$ there is *no transition*.
  - Transition temperature T_c to an $\sim$ 1–2% accuracy
  - Latent heat L, interface tension σ, Higgs field expectation value $v(T)$ in the broken phase, metastability around T_c.
  - (screening) mass spectrum in the broken and the symmetric phases.

- Tools: a *3D effective theory* (dimensional reduction)

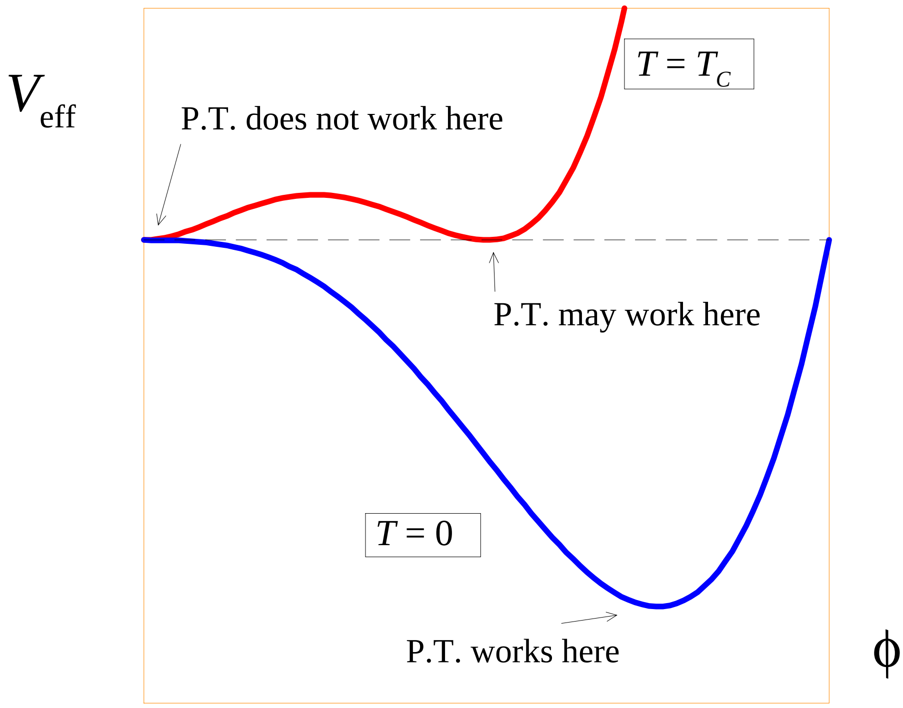
  + a combination of perturbative and non-perturbative methods.

- Result: no room for EW baryogenesis in MSM
- $\longrightarrow$ new physics
- In MSSM: maybe

Analytically:

1. Perturbative analysis:

- Perturbation theory works very well at $T = 0$
- IR regulated by v: $m_W \sim \frac{1}{2} g v$
- At high T, IR divergences appear: symmetric phase?



- 2-loop $V_{\text{eff}}(\phi)$ [Arnold and Espinosa, PRD 47 (1993); Fodor and Hebecker, NPB 432 (1994); Farakos, Kajantie, Rummukainen, Shaposhnikov, NPB 425 (1995); Buchmüller, Fodor and Hebecker, NPB 447 (1995)]
- Small m_H: strong 1st order transition
- Transition becomes weaker when m_H increases (formally 1st order)
- Fails at $m_H \sim 80\,\text{GeV}$ even in broken phase

2. *Non-perturbative RG flow*
[Reuter and Wetterich, NPB 408 (1993);
Bergerhoff and Wetterich, NPB 440 (1995)]:

- Transition 1st order for small m_H
- *No transition* for $m_H \gtrsim 200\,\mathrm{GeV}$ (cross-over)

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3. ε *-expansion* [Arnold and Yaffe, PRD 49 (1994)]:

- $3 + \varepsilon$ dimensions
- Transition becomes weaker when m_H increases, but remains 1st order for all m_H
- Reliability?

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4. *1-loop Schwinger-Dyson equations*
[Buchmüller and Philipsen NPB 443 (1995)]:

- 1st order for $m_H \lesssim 100$ GeV
- *No transition* for $m_H \gtrsim 100$ GeV
- Reliability in the symmetric phase?

Matter–antimatter asymmetry and the electroweak phase transition:

- *Sphaleron* transitions violate $B + L$ number.
- At $T = 0$, the sphaleron rate is strongly suppressed and unobservable [G. t'Hooft, PRL 37 (1976)].
- When $T > T_c$, the rate is not suppressed at all: $\Gamma \propto T^4$.
- $\rightarrow$ Pre-existing asymmetry is washed out at $T > T_c$
- When $T_c > T > 0$, the sphaleron rate is proportional to

$$\Gamma \propto T^4 (E_{\rm sph}/T)^7 \exp[-E_{\rm sph}/T]$$

where the energy of the sphaleron $E_{\rm sph} \approx 7 \times \pi g^{-1} v$.

- **I**. Could the asymmetry have been generated during the electroweak phase transition? [Kuzmin, Rubakov, Shaposhnikov, PLB 155 (1985)]
- **II**. Are sphalerons "frozen" quickly enough after the transition so that the asymmetry is preserved?
- Freezeout condition:

$$E_{\rm sph}(T_c)/T_c > 45 \qquad \rightarrow \qquad v(T_c)/T_c \gtrsim 1.2\,.$$

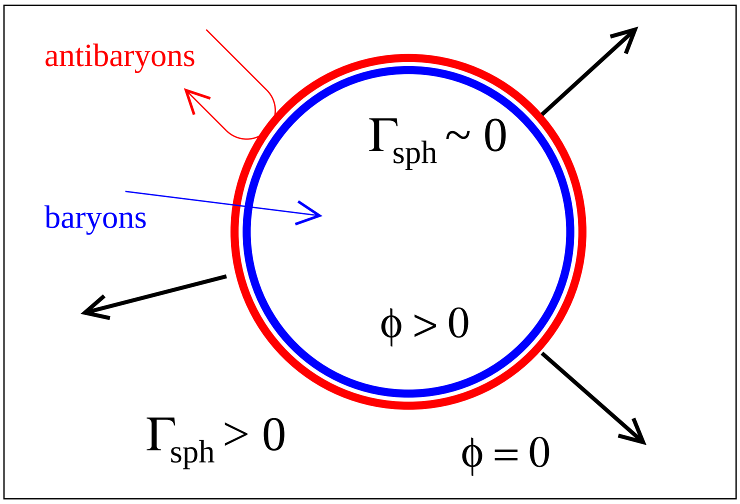
Matter–antimatter asymmetry (= *baryon number* B) generation:

Sakharov conditions [Pisma ZhETF 5 (1967)] must be met:

1. CP and P non-conservation
2. baryon number violation (sphalerons)
3. Out of thermal equilibrium

1. and 2. OK for the electroweak transition, 3. OK if the transition is of first order.

The EW transition occurs at $t \sim 10^{-11}$s, $T \sim 200\,\mathrm{GeV}$. The transition proceeds through *bubble nucleation*:



- Bubble wall lets preferably baryons through $\longrightarrow$ excess of baryons inside, antibaryons outside.
- Inside, $\Gamma_{\mathrm{sph}} \approx 0$, and B is conserved; outside, $\Gamma_{\mathrm{sph}} > 0$, and antibaryons are converted into baryons.
- $\longrightarrow$ excess of matter over antimatter in the Universe.

Mass scales and dimensional reduction:

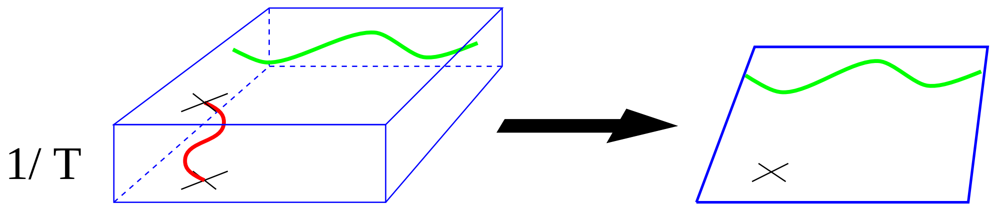
In EW theory near the transition temperature, a wide range of mass scales (weak coupling)

$$\pi T \gg m_D \sim gT \gg g^2 T \quad (\sim m_H(T_c))$$

From perturbative analysis, we know that non-perturbative physics is in light magnetic IR modes $\sim g^2 T$.
$\longrightarrow$ *effective theory* for low-energy modes.

$$\text{Matsubara modes:} \quad \begin{array}{lrcl} \text{Bosons:} & p^2 & = & \vec{p}^{\,2} + (2\pi n T)^2 + m_0^2 \\ \text{Fermions:} & p^2 & = & \vec{p}^{\,2} + (\pi(n+1)T)^2 + m_0^2 \end{array}$$

$\longrightarrow$ all $n \neq 0$ bosonic and *all* fermionic modes $m_3 \sim \pi T$
$\longrightarrow$ only $n = 0$ bosonic modes are light (and independent of $\tau = it$).
$\longrightarrow$ replace the full 4D theory with an *effective* 3D theory of IR-modes by "integrating" out all massive modes



$S_4(\varphi_4) \quad \longmapsto \quad S_3(\varphi_3)$

DR $S_4(\varphi_4) \longrightarrow S_3(\varphi_3)$ can be done *perturbatively*, if

- πT is larger than all relevant mass scales.
- The coupling constants g^2, λ are small (weak coupling).

No IR problems in the derivation of S_3!

Electroweak Lagrangian (Euclidean, 4D):

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + (D_\mu\phi)^\dagger(D_\mu\phi) \\ &+ \frac{1}{2}m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \bar{\psi}(\gamma_\mu D_\mu + g_Y\phi)\psi\end{aligned}$$

Only A^a_i, B_i (spatial gauge fields), ϕ, and A^a_0, B_0 (adjoint 'Higgs' fields) survive 4D$\longrightarrow$3D.

We're interested in the effective theory of scales $\lesssim g^2T$: fields A^a_0 and B_0 have Debye mass $\sim gT$, $g'T$, and can be integrated over.

How to get the effective action?

I. *Ansatz* for (superrenormalizable) Lagrangian

$$\mathcal{L}_3 = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_i\phi)^\dagger(D_i\phi) + \frac{1}{2}m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

II. *Green's function matching:* match the 3D 2,(3),4 -point Green's functions to the corresponding 4D Green's functions to the desired accuracy (We work at 2-loop level for m_3^2 and 1-loop level for couplings).

- Dimensions: $[g_3^2] = [g'^2_3] = [\lambda_3] = \text{GeV}$; field $[\phi] = \text{GeV}^{1/2}$
- Superrenormalizable: g_3, λ_3 do not run; m_3^2 has 1-loop linear and 2-loop log-divergences.

Theory is uniquely fixed by

$$\boxed{g_3, \qquad x \equiv \frac{\lambda_3}{g_3^2} \qquad y \equiv \frac{m_3^2(g_3^2)}{g_3^4} \qquad z \equiv \frac{g'^2_3}{g_3^2}}$$

The simplest example: connection

$$\begin{array}{ccc} \text{4D SU(2)} - \phi & \leftrightarrow & \text{3D SU(2)} - \phi \\ g^2,\ m^2,\ \lambda,\ T^* & \leftrightarrow & g_3^2,\ m_3^2,\ \lambda_3 \end{array}$$

To simplify the expressions, we use here

$$g = 2/3 \qquad m_W = 80.6\,\text{GeV}$$

$$\boxed{\begin{aligned} g_3 &= 0.44015\,T^* \quad (\sim g^2 T) \\ x &= -0.00550 + 2.27196\,\lambda \\ y &= 0.39818 + 2.7981\,\lambda \\ &\quad - 0.6156\,\lambda^2 - 46.45584\,\lambda \left(\frac{m_W}{T^*}\right)^2 \end{aligned}}$$

To give a better intuitive feeling, the results here are often in terms of T^*, m_H^*, where

$$\lambda = \frac{1}{8} g^2 \left(\frac{m_H^*}{m_W}\right)^2$$

NOTE: m_H^* and T^* are only 'tree-level' T, m_H: in order to get physical values, pole m_H, m_W are needed (for a particular 4D model). For 4D SU(2)-Higgs the differences are small.

Lattice action

SU(2) + Higgs in 3D:

$$L_3 = \frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\phi)^\dagger(D_i\phi) + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

- Dimensions: $[\phi] = \mathrm{GeV}^{1/2}$, $[g_3^2] = [\lambda_3] = \mathrm{GeV}$
- Theory is fixed by

$$g_3 \qquad x \equiv \lambda_3/g_3^2 \qquad y \equiv m_3^2(g_3^2)/g_3^4$$

Lattice: $(\Phi^2 \equiv \frac{1}{2}\mathrm{Tr}\,\phi^\dagger\phi)$:

$$\boxed{S = \beta_G \sum_\square (1 - \frac{1}{2}\mathrm{Tr}\,U_\square) - \beta_H \sum_{x,i} \frac{1}{2}\mathrm{Tr}\,\phi_x^\dagger U_{x,i}\phi_{x+i} + \sum_x [\Phi^2 + \beta_R(\Phi^2 - 1)^2]}$$

CPC (exact when $a \to 0$):

$$\begin{aligned}
g_3^2 a &= \frac{4}{\beta_G}, \\
x &= \frac{1}{4}\lambda_3 a \beta_G = \frac{\beta_R\beta_G}{\beta_H^2}, \\
y &= \frac{\beta_G^2}{8}\left(\frac{1}{\beta_H} - 3 - \frac{2x\beta_H}{\beta_G}\right) + \frac{9.5277\beta_G}{32\pi}(1+4x) + \\
&\quad + \frac{1}{16\pi^2}\left[\left(\frac{51}{16} + 9x - 12x^2\right)\left(\ln\frac{3\beta_G}{2} + 0.09\right) + 5.0 + 5.2x\right].
\end{aligned}$$

4D actions:
- SU(2) - Higgs
- + fermions
- + U(1)
- MSSM ...

Fix physics

Physical quantities at $T = 0$
perturbatively
(pole masses, g_R)

'integration' over scale T by 2–loop matching
integrate over A_0 (gT)

3D continuum SU(2) + ϕ (+ U(1))
parameters x, y (+ z)

2–loop, *exact* connection
in the continuum limit

3D lattice SU(2) + ϕ (+ U(1))

K. Farakos, K. Kajantie, K. Rummukainen, M. Shaposhnikov,
NPB 425 (1994); NPB 442 (1995);
K.K., M. Laine, K.R., M.S, NPB 458 (1996); U(1): hep-lat/9612006
3D continuum – lattice: M. Laine, NPB 451 (1995)
MSSM: M. Laine, NPB 481 (1996); J.Cline, K.Kainulainen, NPB 482 (1996);
M.Losada, hep-ph/9605266
SU(5) GUT: A. Rajantie, hep-ph/9702255

Features:

Encapsulation:

1. Perturbative dimensional reduction 4D $\leftrightarrow$ 3D
2. Analysis of the 3D effective action (analytical/numerical)
3. 4D theory $\leftrightarrow$ $T = 0$ physical observables

Reduction:

4D couplings $+ T \quad \mapsto \quad x,\, y\ (z)$

$y \sim T$: 4D theories are characterized by the value of x.

Economy:

3D theory simple to analyze numerically/analytically

Numerically:

- 4D fermions have *only* massive $\sim \pi T$ modes
  $\longrightarrow$ effective action is bosonic
- Heavy scales and 4th dimension missing
  $\longrightarrow$ easy on computers
- Superrenormalizability
  $\longrightarrow$ scaling becomes transparent (UV-sector is in control)

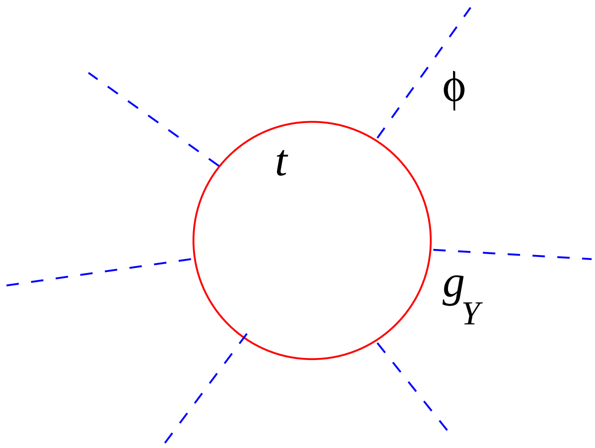
Validity:

- Error in effective action:

$$\delta S = \mathcal{O}(m_\alpha^2(T)/T^2) \qquad (+\text{const.})$$

- Weak couplings: $g^2 \ll 1, \;\; \lambda \ll 1$
  $\longrightarrow m_H \lesssim 240\,\text{GeV}$ (MSM)
- Broken phase: $m_W(T_c) \approx \frac{1}{2} g v_c \ll T$
  $\longrightarrow m_H \gtrsim 30\;\; \text{GeV}$
- Action does not contain Dim-6 -operators (which would still give a renormalizable action in 3D)

For example, a fermion loop causes a shift $\sim 1\%$ in minimum location of $V_{\text{eff}}(\phi)$ (top quark).



Conclusions

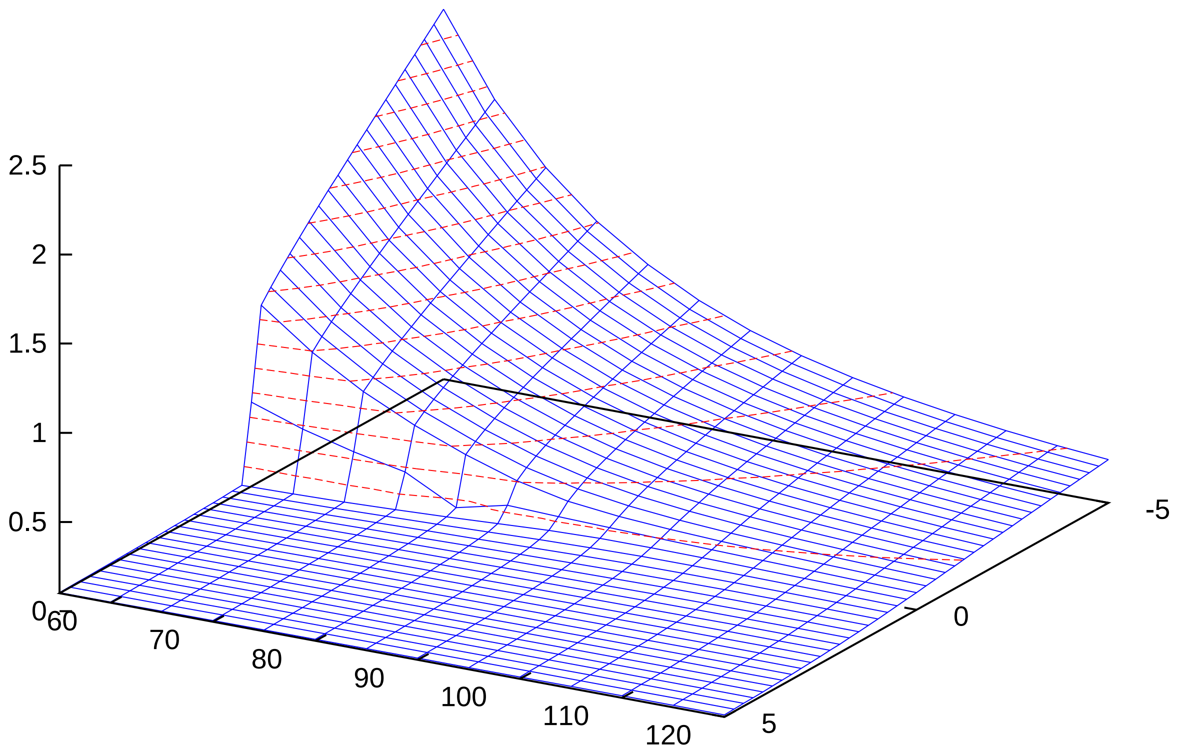
In MSM:

- $m_H^* \lesssim 75\,\text{GeV}$ ($x \lesssim 0.10$) transition is 1st order and turns into a cross-over for larger m_H
- $T_c \sim 1\,\text{GeV}$ accuracy
- latent heat L, interface tension σ, metastability temperatures, discontinuity in $< \phi >$
- 2-loop P.T. provides a good guideline for the EW transition for $m_H^* \lesssim 60\,\text{GeV}$ ($x \lesssim 0.09$). However, deviations are seen:
  - $T_c^{\text{latt}} < T_c^{\text{pert}}$ (difference 1–2%)
  - $\sigma^{\text{latt}} \sim \frac{1}{4}\sigma^{\text{pert}}$
- Phases analytically connected, but they behave like real 'symmetric' and 'broken' phases to a very good accuracy
- confinement in the high-temperature phase
- Chern-Simons diffusion?
- EW baryogenesis requires $x < 0.04$. *No m_H-value can satisfy this in MSM.* MSSM?

3D effective action:

accurate and very economical method for the phase transition studies in *weakly coupled* systems (MSM, MSSM, GUT?).

$< \Phi^2 > /g_3^2$ for $m_H = 55 - 120\,\mathrm{GeV}$



U1 gauge field survives DR, and it can cause significant non-perturbative effects. 3D action is

$$\mathcal{L}_3 = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_i\phi)^\dagger(D_i\phi) + \frac{1}{2}m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

where

$$D_i = \partial_i + ig_3A_i + \frac{1}{2}ig'_3B_i$$

3 dimensionless parameters:

$$x \equiv \frac{\lambda_3}{g_3^2} \qquad y \equiv \frac{m_3^2(g_3^2)}{g_3^4} \qquad z \equiv \frac{g'^2_3}{g_3^2} = \tan^2\theta_W$$

We use:

$z = 0.3 \approx m_Z^2/m_W^2 - 1$

$x = 0.0644$ and 0.6240 ($m_H^* = 60$ and $180\,\mathrm{GeV}$)

Mass spectrum: $m_H,\ m_{W^\pm},\ m_Z,\ m_\gamma$

The effect of U(1) field when $x = 0.0644$

		$z = 0$	$z = 0.3$
y_c	lattice	-0.00142(36)	0.00724(45)
	+ pert. U(1)		0.0060
	2-loop P.T.	0.01141	0.01882
$\Delta\ell_3$	lattice	0.471(8)	0.569(17)
	+ pert. U(1)		0.55
	2-loop	0.493	0.575
σ_3	lattice	0.0116(28)	0.0165(30)
	+ pert. U(1)		0.014
	2-loop	0.0401	0.0487