Cooperative and coalition games

- 1. Non-cooperative games

Here

2. Cooperative game

3. Partition function game

2. A Cooperative Game

- Traditionally, cooperative games are modelled as characteristic function games (c-games).
- In these games it is assumed that different coalitions between the players can be formed.
- Consider an fish stock harvested by three countries (1,2 and 3).
- In terms of coalitions, the following alternatives are possible: no agreement, bilateral agreements and trilateral agreement.

2. A Cooperative Game (cont.)

- Consider the following payoffs for each coalition:

Coalition (K)	Payoff (π)
(1,2,3) – Grand Coalition	10,000
(1,2) – Two-player Coalition	6,000
(1,3) – Two-player Coalition	5,000
(2,3) – Two-player Coalition	3,000
(1) – Singleton	1,200
(2) – Singleton	800
(3) – Singleton	500

- Let us compute the surplus of each coalition

2. A Cooperative Game (cont.)

Coalition (K)	Payoff (π)	CS (K)
(1,2,3) – Grand Coalition	10,000	7,500
(1,2) – Two-player Coalition	6,000	4,000
(1,3) – Two-player Coalition	5,000	3,300
(2,3) – Two-player Coalition	3,000	1,700
(1) – Singleton	1,200	0
(2) – Singleton	800	0
(3) – Singleton	500	0

CS(K): payoff of the coalition K subtracted by the payoffs of its members as singletons (“threat points”).

2. A Cooperative Game (cont.)

- Finally, let's normalise the values:

Coalition (K)	Payoff (π)	CS (K)	v(K)
(1,2,3) − Grand Coalition	10,000	7,500	1
(1,2) − Two-player Coalition	6,000	4,000	0.53
(1,3) − Two-player Coalition	5,000	3,300	0.44
(2,3) − Two-player Coalition	3,000	1,700	0.23
(1) − Singleton	1,200	0	0
(2) − Singleton	800	0	0
(3) − Singleton	500	0	0

Where $v(K)=CS(K)/CS(1,2,3)$

2. A Cooperative Game (cont.)

- The characteristic function assigns a value to each possible coalition.
- Thus, the table shows the values of characteristic function for this fishery.
- A game in characteristic form can be denoted by (M,v), where M represents the set of all possible coalitions and v the characteristic function.
- A central issue in cooperative games is how to divide the gains from cooperation in a “fair” way.
- The most common “fair” sharing rules used in the c-game are the Nash bargaining solution and the Shapley value.

2. A Cooperative Game (cont.)

- The Shapley Value is based on the average contribution that each member makes to the set of possible coalitions (Kaitala and Lindroos 1998).

Definition: The Shapley value is the imputation $Z = (Z_1, Z_2, \ldots, Z_n)$, $Z_i \geq 0$ and $\sum_{i=1}^{n} Z_i = 1$, given by:

$$Z_i = \sum_{K \subseteq M} \left[v(K) - v(K \setminus \{i\}) \right] \frac{(k-1)!(n-k)!}{n!}$$

K - includes all coalitions to which player i belongs;
k - number of players of coalition K;
M - set of all possible coalitions;
n - total number of players.

2. A Cooperative Game (cont.)

- Thus Shapley Value distributes the cooperative benefits according to the marginal contributions of each player.

- These games have been introduced in the fisheries literature by:

  Kaitala, V. and Lindroos, M. (1998) “Sharing the Benefits of Cooperation in High Sea Fisheries: A Characteristic Function Game Approach”, *Natural Resource Modeling* 11: 275–99.

3. Partition Function Games

- The framework of a characteristic function approach, although sufficiently general to encompass many contributions of coalition formation theory, is not fully satisfactory (Greenberg, 1994).
- Most importantly, it ignores the possibility of externalities among coalitions, that is, the effects that coalition mergers have on the payoffs of players who belong to the other coalitions.
- Definition: a positive (negative) externality occurs when a merger of coalitions increases (decreases) the payoff of a player belonging to a coalition not involved in the merger.

3. Partition Function Games

- In the context of straddling fish stocks management, through regional fisheries management organisations, positive externalities are generally present.
- As these organizations tend to adopt conservative management strategies, non-members are typically better off when more players become members, as free rider strategies can be adopted.
- The formation of economic coalitions with externalities has opened a new strand of literature on non-cooperative game theory (Yi, 1997).

3. Partition Function Games

- Most studies are centred on finding the equilibrium number and size of coalitions and share a common two-stage game framework.
- In the first stage players form coalitions, and in the second-stage coalitions engage in noncooperative behaviour.
- The coalition payoffs in the second stage are defined as a partition function. This function assigns a value to each coalition, which depends on the entire coalition structure.

3. Partition Function Games

- Partition Function Games were introduced by Thrall and Lucas (1963) but was only revived in the 1990s by authors such as Yi, Bloch, Ray and Vohra.
- The first applications to fisheries are due to Pintassilgo (2003) and Pham Do and Folmer (2006).
- A good survey on recent partition function games applied to economics:

  Yi, Sang-Seung (2003). Endogenous Formation of Economic Coalitions: a Survey of the Partition Function Approach, in Carraro, Carlo (eds.), *The Endogeneous Formation of Economic Coalitions*. The Fondazione Eni Enrico Mattei (FEEM) Series on Economics and the Environment, Edward Elgar.

3.1 Coalition Formation in Straddling Stock Fisheries: A Partition Function Approach

- Let us introduce a partition function game on a straddling stock fishery based on:

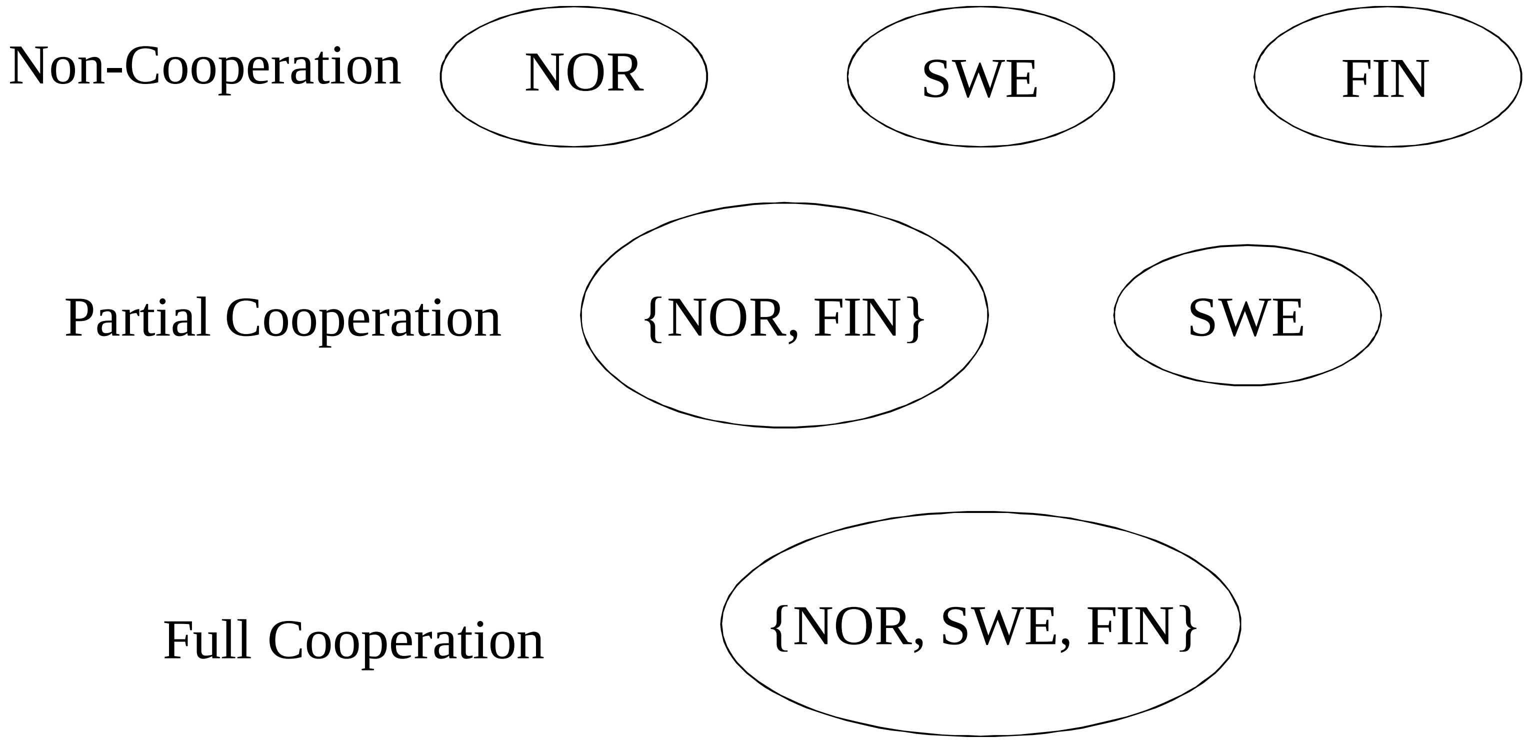
  Pintassilgo, P. and M. Lindroos (2007). Coalition Formation in Straddling Stock Fisheries: A Partition Function Approach. *International Game Theory Review*.

- The paper models straddling stock fisheries through a game in partition function form using the classical Gordon-Schaefer bioeconomic model.

3.1 Coalition Formation in Straddling Stock Fisheries: A Partition Function Approach

- The following elements are analysed: the existence of positive externalities, the stability of coalition structures and the equilibrium.
- Moreover, the results are used to shed light on the prospects of cooperative resource management as prescribed by the UN Fish Stocks Agreement.

Coalitional games: Searching for equilibrium cooperation structures



The Bioeconomic Model

- *n ex-ante* symmetric players (fleets/ countries)

- Fish stock dynamics

$$\frac{dX}{dt} = G(X) - \sum_{i=1}^{n} H_i$$

$$G(X) = rX\left(1 - \frac{X}{k}\right)$$

$$H_i = qE_iX$$

X - fish stock biomass;

$G(X)$ - stock growth function;

H_i - harvest of player i;

E_i - fishing effort of player i.

- Steady-state relation between fishing effort and stock:

$$X = \frac{k}{r}\left(r - q\sum_{i=1}^{n} E_i\right)$$

The Bioeconomic Model

- The aggregate *Economic Rent* from the fishery is

$$ER = p\sum_{i=1}^{n} H_i - c\sum_{i=1}^{n} E_i$$

p - price

c - cost per unit of effort

The Game

- Assume that a Regional Fishery Management Organisation (RFMO) is established with the purpose of managing and conserving a given straddling fish stock.

- Two-Stage Game
  - Single Coalition and Open-Membership Game (d'Aspremont et al., 1983).
  - First stage: players choose membership (RFMO versus no RFMO);
  - Second stage: players choose fishing efforts that maximise the steady-state rent from the fishery, given the behaviour of the others.

The Game

- The coalition payoffs in the second stage are defined as a partition function.
- Symmetric players: equal sharing of coalition payoffs assumed.
- Game solved by backward induction.

Partition Function

- The Per-member Partition Function was computed for all coalition structures:

Full Cooperation

The grand coalition solves the following problem:

$$\underset{E}{Max}\,\Pi = pH - cE = pqE\frac{k}{r}\left(r - qE\right) - cE$$

Where $H = \sum_{i=1}^{n} H_i$ and $E = \sum_{i=1}^{n} E_i$

What is the solution of this problem ?

Partition Function

$$E^{*} = \frac{r}{2q}(1 - b)$$

Where $b = \dfrac{c}{pqk} \in [0;1]$

Usually referred as "inverse efficiency parameter".

The stock level is:

$$X^{*} = \frac{1}{2}\left(k + \frac{c}{pq}\right)$$

Each member of the grand coalition receives the following payoff:

$$p\left(n;\{n\}\right) = \frac{1}{n}\left(pqk - c\right)\frac{r}{4q}(1 - b)$$

Partition Function

Non - Cooperation

Consider a generic coalition structure with two or more coalitions:

$$C = \{1, \ldots, 1, n - m\}$$

Where:

1 - represents a singleton

$n - m$ - the RFMO coalition if $n - m \geq 2$

Each of the $m+1$ coalitions solves the same problem.

The problem of a given coalition j can be represented as:

$$\underset{E_j}{Max}\, \pi_j = pH_j - cE_j = pqE_j \frac{k}{r}\left(r - q\left(E_j + mE_{nj}\right)\right) - cE_j$$

Where:

H_j and E_j denote the harvest and the fishing effort of coalition j, respectively;

E_{nj} - fishing effort of any coalition other than j.

Partition Function

- Compute the reaction function of each coalition.
- What is the fishing effort of each coalition at the Nash equilibrium ?
- What is the equilibrium stock level ?
- What are the coalitions' payoffs ?

$$\pi\left(1;\{1,\ldots,1,n-m\}\right)=\frac{(pqk-c)\,r\,(1-b)}{(m+2)^2\,q}$$

$$\pi\left(n-m;\{1,\ldots,1,n-m\}\right)=\frac{(pqk-c)\,r\,(1-b)}{(n-m)(m+2)^2\,q}$$

Partition Function

- The game is characterised by Positive Externalities:

If coalitions merge to form a larger coalition, the coalitions not involved in the merge are better off.

Stability and Equilibrium Coalition Structures

According to Yi (1997), in the context of positive externalities the concept of stand-alone stability (or internal stability) is particularly useful, namely in characterizing equilibrium coalition structures.

Definition

A coalition structure is stand-alone stable if and only if no player finds it profitable to leave his coalition to form a singleton coalition, holding the rest of the coalition structure constant.

Result 1

In this fishery game, the grand coalition is stand-alone stable if and only if the number of players is two.

Stability and Equilibrium Coalition Structures

Result 2

The only coalition structure, with more than one coalition, that is stand-alone stable is the one formed by singletons.

Result 3

The Nash equilibrium coalition structure of the game is:

$$C^{NE} = \begin{cases} \{2\} \Leftarrow & n = 2 \\ \{1,\ldots,1\} \Leftarrow & n \geq 3 \end{cases}$$

Conclusion

- Using a two-stage game the paper shows that, apart from the case of two players, the grand coalition is not a Nash equilibrium outcome. Furthermore, in the case of three or more players, the only Nash equilibrium coalition structure is the one formed by singletons.

- These results are in line with previous studies using two-stage partition function games.

Conclusion

- Yi (1997) concludes that for classical examples of positive externalities, such as output cartels and coalitions formed to provide public goods, an open membership game rarely supports the grand coalition as a Nash equilibrium, and equilibrium coalition structures are often very fragmented.

- Pintassilgo (2003), using a complex bio-economic model, shows that for the Northern Atlantic bluefin Tuna fishery there's no sharing rule that makes the grand coalition stable.

Conclusion

- According to these results the prospects of cooperation in straddling stock fisheries are low if countries can free-ride cooperative agreements.

- Thus, in order to protect cooperation the legal regime must prevent those who engage in non-cooperative behaviour from having access to the resource.

3.2 Extending the Game to Heterogeneous Players

- Pintassilgo, P., M. Finus, M. Lindroos and G. Munro (2010). Stability and Success of Regional Fisheries Management Organizations. Environmental and Resource Economics.

- Assume that players can differ in harvesting costs.

  - Profit of State i:

$$\Pi_i = pH_i - c_iE_i = pqE_iX^* - c_iE_i$$

p - is the price for fish;

c_i - cost per unit of effort of state i.

- Two-stage game solved by backward induction.

First Stage

- A coalition is stable iff no signatory has incentive to leave it (internal stability) and no singleton has incentive to join it (external stability).

Internal Stability: $\Pi_i(S) \geq \Pi_i(S \setminus \{i\}) \quad \forall\, i \in S$

External Stability: $\Pi_j(S) \geq \Pi_j(S \cup \{j\}) \quad \forall\, j \notin S$

- Potential internal stability (Eyckmans and Finus, 2004)

$$\Pi_S(S) \geq \sum_{i \in S} \Pi_i(S \setminus \{i\})$$

- In our game this condition depend only on:

n - *number of fishing states*

m - *number of coalition members*

$b_i = \dfrac{c_i}{pqk} \in [0,1],\ i \in N$ - inverse efficiency parameter

Simulation Method

- Assumption:

$$b_i \sim U(0,1), \quad \forall\, i \in \{1,\ldots,n\}$$

- Monte Carlo simulation method: 50,000 simulations of the b vector, for each combination of n and m.
  - Stability likelihood (probability).
  - Social Gain Index (SGI)
  - Closing the Gap Index (CGI)

$$SGI(b,n) = \frac{A\Pi(N) - A\Pi\left(1_{(n)}\right)}{A\Pi(N)}$$

$$CGI\left(S_j^*(b),n\right) = \frac{A\Pi\left(S_j^*\right) - A\Pi\left(1_{(n)}\right)}{A\Pi(N) - A\Pi\left(1_{(n)}\right)}$$

$A\Pi(N)$ - Aggregate payoff of the grand coalition

S_j^* - a given stable coalition

$\sum_{i=1}^{n} \Pi_i\left(1_{(n)}\right)$ - Aggregate payoff when all players are singletons.

Simulation Results

- Potential Internal Stability Likelihood

Number of Coalition Members (m)	Number of Players (n)								
	2	3	4	5	6	7	8	9	10
n	1	0.777	0.345	0.103	0.022	0.004	0.001	0	0
n-1	1	0.826	0.417	0.147	0.037	0.007	0.001	0	0
n-2	–	1	0.646	0.273	0.080	0.019	0.004	0.001	0
n-3	–	–	1	0.538	0.195	0.054	0.011	0.002	0.001
n-4	–	–	–	1	0.466	0.150	0.037	0.007	0.002
n-5	–	–	–	–	1	0.409	0.120	0.026	0.005
n-6	–	–	–	–	–	1	0.367	0.098	0.021
n-7	–	–	–	–	–	–	1	0.333	0.081
n-8	–	–	–	–	–	–	–	1	0.308
n-9	–	–	–	–	–	–	–	–	1

Simulation Results

- Stability Likelihood – Base Case

		Number of Players (*n*)								
		2	3	4	5	6	7	8	9	10
Number of Coalition Members (*m*)	n	1	0.777	0.345	0.103	0.022	0.004	0.001	0	0
	n-1	0	0.127	0.149	0.074	0.023	0.004	0.001	0	0
	n-2	–	0.028	0.148	0.101	0.036	0.010	0.002	0	0
	n-3	–	–	0.036	0.126	0.071	0.023	0.005	0.001	0
	n-4	–	–	–	0.035	0.112	0.053	0.015	0.003	0.001
	n-5	–	–	–	–	0.030	0.101	0.040	0.011	0.002
	n-6	–	–	–	–	–	0.026	0.090	0.031	0.007
	n-7	–	–	–	–	–	–	0.025	0.082	0.025
	n-8	–	–	–	–	–	–	–	0.020	0.076
	n-9	–	–	–	–	–	–	–	–	0.017
$\overline{SGI}(n)$		14.8	25.5	33.6	39.9	45.1	49.4	53.0	56.1	58.8
$\overline{CGI}(n)$		100	87.2	55.4	30.4	16.3	9.3	5.9	4.0	2.8

Simulation Results

- Stability Likelihood – Base Case

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Simulation Results

- Success Indexes Estimates – Asymmetry Effect

Range of b_i´s		Number of Players								
		2	3	4	5	6	7	8	9	10
		Asymmetry Effect								
0.2	$\overline{SGI}(n)$	11.1	25.0	36.0	44.4	51.0	56.3	60.5	64.0	66.9
	$\overline{CGI}(n)$	100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
[0.1;0.3]	$\overline{SGI}(n)$	16.1	29.9	39.2	45.6	50.1	53.5	56.2	58.7	60.8
	$\overline{CGI}(n)$	100.0	43.8	17.3	9.9	6.6	4.7	3.5	2.7	2.1
[0;0.4]	$\overline{SGI}(n)$	17.3	28.2	35.5	41.2	45.9	49.9	53.3	56.3	59.0
	$\overline{CGI}(n)$	100.0	76.7	42.7	23.3	13.5	8.4	5.5	3.9	2.8

Simulation Results

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	$\overline{CGI}(n)$	100.0	76.7	42.7	23.3	13.5	8.4	5.5	3.9	2.8

Simulation Results

- Success Indexes Estimates – Efficiency Effect

Range of b_i´s		Number of Players								
		2	3	4	5	6	7	8	9	10
		Efficiency Effect								
[0;0.2]	$\overline{SGI}(n)$	15.8	29.7	39.4	46.2	50.9	54.4	57.2	59.5	61.6
	$\overline{CGI}(n)$	100	38.9	15.2	8.5	5.7	4.1	3.1	2.4	1.9
[0.2;0.4]	$\overline{SGI}(n)$	16.4	29.9	38.8	44.8	49.0	52.4	55.3	57.9	60.2
	$\overline{CGI}(n)$	100.0	50.2	19.8	11.4	7.6	5.4	4.0	3.0	2.3
[0.4;0.6]	$\overline{SGI}(n)$	17.2	29.3	36.9	42.3	46.8	50.6	53.8	56.7	59.2
	$\overline{CGI}(n)$	100	67.8	31.6	17.6	11.1	7.3	5.0	3.6	2.6

Simulation Results

- Success Indexes Estimates – Efficiency Effect

Range of b_i´s		Number of Players								
		2	3	4	5	6	7	8	9	10
		Efficiency Effect								
[0;0.2]	$\overline{SGI}(n)$	15.8	29.7	39.4	46.2	50.9	54.4	57.2	59.5	61.6
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Simulation Results

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[0.2;0.4]	$\overline{SGI}(n)$	16.4	29.9	38.8	44.8	49.0	52.4	55.3	57.9	60.2
	$\overline{CGI}(n)$	100.0	50.2	19.8	11.4	7.6	5.4	4.0	3.0	2.3
[0.4;0.6]	$\overline{SGI}(n)$	17.2	29.3	36.9	42.3	46.8	50.6	53.8	56.7	59.2
	$\overline{CGI}(n)$	100	67.8	31.6	17.6	11.1	7.3	5.0	3.6	2.6

Conclusion

- It is shown that the paradox of the global commons (Barrett, 1994) also applies to international fisheries.
- The higher the number of players the higher are the gains from cooperation but the lower is the success of coalition formation.
- The prospects of stable cooperative agreements increase with players' cost asymmetry and decrease with the overall efficiency level.