Two-player game

NMA course

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Non-cooperative games

- Individual strategies for the players
- Reaction functions, best reply
- Nash equilibrium definition

- Stages games at different levels

- Repeated games, folk theorems, sustaining cooperative behaviour as equilibria

- Dynamic games

Why non-cooperative

- Classification: strategic (static), extensive (dynamic), coalition
- Important in fisheries non-cooperation (competition) vs cooperation
- Division not clear, almost all games have both non-cooperative and cooperative elements
- Typically in economics non-cooperative game theory dominates

What are non-cooperative games about

- How fisher’s decisions interact with other fishers’ decisions

- What is the best strategy for the fishers

- What is exected to happen is the fishery? Depends on rules of the game, number of players, biological factors

- Why fishers behave as they do?

- Assume rational choice

International fisheries negotiations

Nature of negotiations

- Countries attempt to sign and ratify agreements to maximise their own economic benefits

- Negotiations typically time-consuming

- Agreements not binding à self-enforcing or voluntary agreements

Explaining the tragedy of the commons

- Can we explain the seemingly irrational behaviour in the world’s fisheries, overexploitation, overcapitalisation, bycatch…
- Non-cooperative game theory explains this behaviour
- Non-cooperative games vs open access (freedom of the seas)

Nash equilibrium

- Each player chooses the best available decision

- It is not optimal for any single player to unilaterally change his strategy

- There can be a unique equilibrium, multiple equilibria or no equilibria

A two-player non-cooperative fisheries game

- Assume there are *n* players (fishers, fishing firms, countries, groups of countries) harvesting a common fish resource *x*
- Each player maximises her own economic gains from the resource by choosing a fishing effort E_i
- This means that each player chooses her optimal e.g. number of fishing vessels taking into account how many the other players choose
- As a result this game will end up in a Nash equilibrium where all individual fishing efforts are optimal

Building objective functions of the players

- Assume a steady state:

$h_i = qE_i x$

$$\frac{dx}{dt} = F(x) - \sum_{i=1}^{n} h_i = 0$$

- By assuming logistic growth the steady state stock is then

Stock biomass depends on all fishing efforts

$$x = K\left(1 - \frac{q\sum_{i=1}^{n} E_i}{R}\right)$$

Objective function

- Players maximise their net revenues (revenues – costs) from the fishery
- max $ph_i - c_i E_i$
- Here p is the price per kg, h_i is harvest of player i, c_i is unit cost of effort of player i

Two maximisation problems

- max $\quad p_1 = ph_1 - cE_1 \;\&\; p_2 = ph_2 - cE_2$

$$p_1 = pqE_1K(1 - \frac{q(E_1 + E_2)}{R}) - cE_1$$

FOC à reaction function

$$\frac{\partial p_1}{\partial E_1} = pqK - \frac{2pq^2E_1K + pq^2E_2K}{R} - c = 0$$

$$-\frac{2pq^2E_1K}{R} = c - pqK + \frac{pq^2E_2K}{R}$$

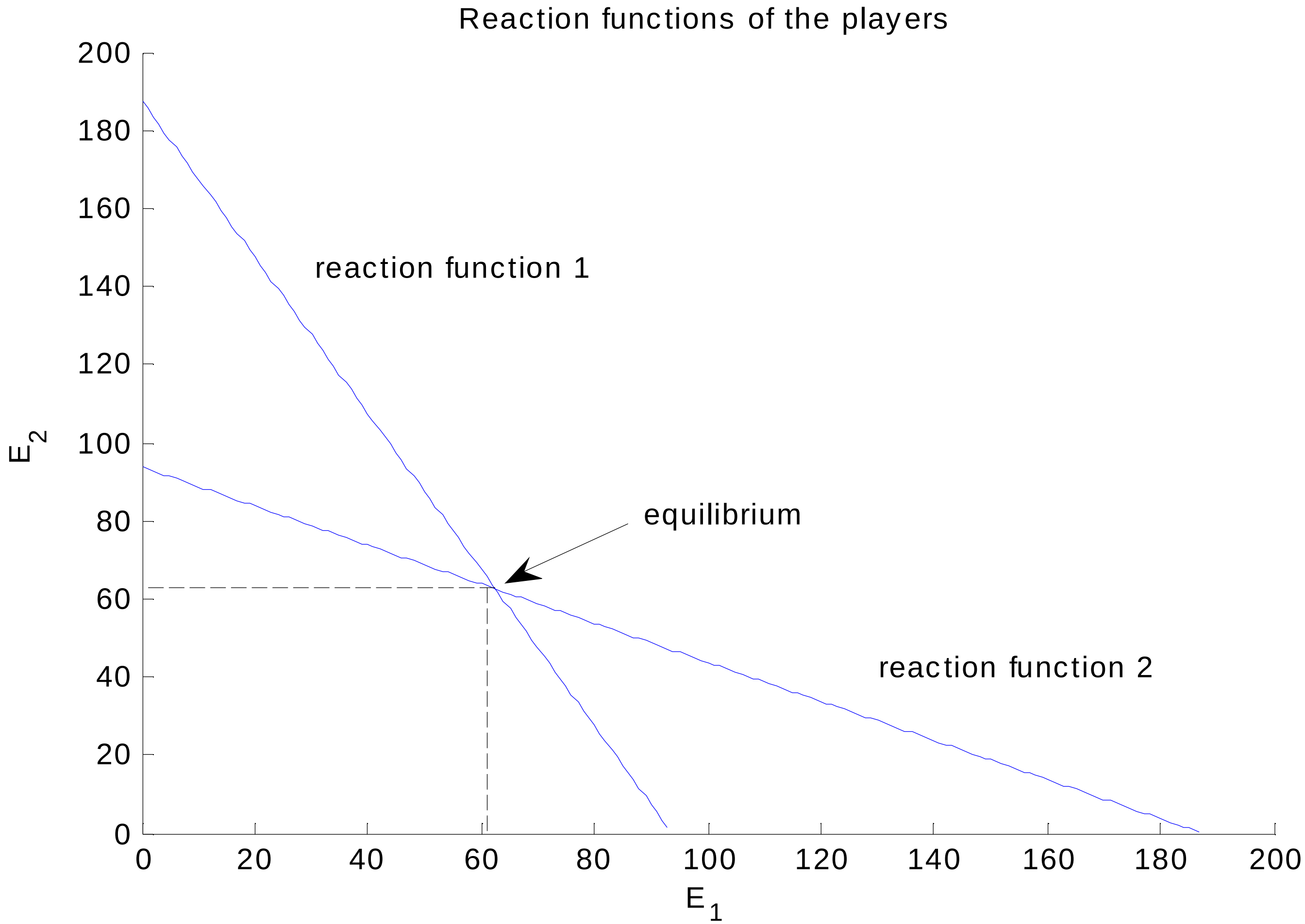
$$\Rightarrow \; E_1 = -\frac{cR}{2pq^2K} + \frac{pqKR}{2pq^2K} - \frac{pq^2E_2K}{2pq^2K}$$

Country 1 reaction function

$$\boxed{E_1 = \frac{R}{2q}(1 - b) - \frac{E_2}{2}}$$

Nash Equilibrium

- At the intersection of the reaction functions



Equilibrium fishing efforts

$$E_1 = \frac{R}{3q}(1 - b) = E_2$$

$$E_{tot} = \frac{2R}{3q}(1 - b)$$

Equilibrium fishing efforts for n asymmetric players

- Derive by using the *n* reaction curves

$$E_i = \frac{nR}{(n+1)q}(1 - b_i) - \sum_{j \neq i}^{n-1} \frac{R}{(n+1)q}(1 - b_j)$$

- The equilibrium fishing efforts depend on the efficiency of all players and the number of players

Illustration

- Nash-Cournot equilibrium
- Symmetric case
- Schäfer-Gordon model

Exercises

- Compute the symmetric 3-player and *n* player equilibrium. First solve 3-player game, then extend to n players. Compute equilibrium efforts and stock in both cases.
- Compute the 3-player and n-player asymmetric equilibrium. Compute equilibrium efforts in both cases.

A two-stage game (Ruseski JEEM 1998)

- Assume two countries with a fishing fleet of size n_1 and n_2
- In the first stage countries choose their optimal fleet licensing policy, i.e., the number of fishing vessels.
- In the second stage the fishermen compete, knowing how many fishermen to compete against

- The model is solved backwards, first solving the second stage equilibrium fishing efforts
- Second, the equilibrium fleet licensing policies are solved

Objective function of the fishermen

- The previous steady state stock is then

$$x = K\left(1 - \frac{q(E_1 + E_2)}{R}\right)$$

$$\text{where } E_1 = e_{1v} + \sum_{w \neq v}^{n_1 - 1} e_{1w}$$

- The individual domestic fishing firm v maximises

$$\max \; pqe_{1v}x - ce_{1v}$$

Reaction functions

- In this model the domestic fishermen compete against domestic vessels and foreign vessels
- The reaction between the two fleets is derived from the first-order condition by applying symmetry of the vessels

$$pqK - \frac{2pq^2 e_{1v}K + \sum_{w \neq v}^{n_1 - 1} pq^2 e_w K + pq^2 E_2 K}{R} - c = 0$$

$$E_1 = \frac{n_1}{n_1 + 1}\left[\frac{R}{q}(1 - b) - E_2\right] = n_1 e_{1v}$$

Equilibrium fishing efforts

- Analogously in the other country

$$E_2 = \frac{n_2}{n_2+1}[\frac{R}{q}(1-b) - E_1]$$

- By solving the system of two equations yields the equilibrium

$$E_2 = \frac{Rn_2}{q}(\frac{1-b}{1+n_1+n_2})$$

$$E_1 = \frac{Rn_1}{q}(\frac{1-b}{1+n_1+n_2})$$

Equilibrium stock

- Insert equilibrium efforts into steady state stock expression

- The stock now depends explicitly on the number of the total fishing fleet

$$x = \frac{K[1+(n_1+n_2)b]}{1+n_1+n_2}$$

Equilibrium rent

- Insert equilibrium efforts and stock into objective function to yield

$$P_1 = RpK \frac{n_1(1-b)^2}{(1+n_1+n_2)^2}$$

First stage

- The countries maximise their welfare, that is, fishing fleet rents less management costs

$$\max W_1 = P_1 - n_1 F$$

- The optimal fleet size can be calculated from the FOC (implicit reaction function)

$$\frac{\partial W_1}{\partial n_1} = RpK \frac{(1 - n_1 + n_2)(1 - b)^2}{(1 + n_1 + n_2)^3} - F = 0$$

Results

- Aplying symmetry and changing variable m = 2n+1

$$\frac{RpK(1-b)^2}{(1+n)^3} - F = 0$$

$$n_1 = \frac{1}{2}\left\{\left[\frac{RpK(1-b)^2}{F}\right]^{1/3} - 1\right\}$$

- With F=0 à open access

Discussion

- Subsidies
- Quinn & Ruseski: asymmetric fishermen
- entry deterring strategies: Choose large enough fleet so that the rival fleet is not able make profits from the fishery
- Kronbak and Lindroos ERE 2006 4 stage coalition game