Clark-Munro -model

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JEEM 1975

- Crutchfield & Zellner (1962)
- Turvey (AER 1964)
- Complicated dynamic optimisation models (calculus of variations)
- Clark & Munro integrated capital theory to the theory of renewable resources using optimal control theory
- IIFET 2016 keynote talk in Aberdeen on this paper

Biological constraint

- Change in the fish stock is defined by biological growth function F(x) less harvests.
- Equation of motion will therefore include a term depending on the state variable (cf Hotelling model).

Optimal control problem

- Max J= $\int_0^{\infty} e^{-rt}\left[p - c(x)\right]h(t)\,dt$

- st $\dot{x} = F(x) - h(t)$

Assumptions

- There is an upper limit for the fishing fleet Emax.

- Higher stock means smaller costs:

  $c'(x) < 0$

Current value Hamiltonian:

$$H = [p - c(x)]h(t) + \mu(t)[F(x) - h(t)]$$

$$\frac{\partial H}{\partial h} = p - c(x) - \mu(t) = 0$$

$$\Rightarrow \quad \dot{\mu} = -c'(x)\dot{x} = -c'(x)[F(x) - h(t)]$$

Dynamic condition & solving

$$-\frac{\partial H}{\partial x} = c'(x)h(t) - \mu(t)F'(x) = \dot{\mu}(t) - r\mu(t)$$

$$c'(x)h(t) - [p - c(x)]F'(x) = -c'(x)[F(x) - h(t)] - r[p - c(x)]$$

$$-[p - c(x)]F'(x) = -c'(x)F(x) - r[p - c(x)]$$

Golden rule of renewable resource use

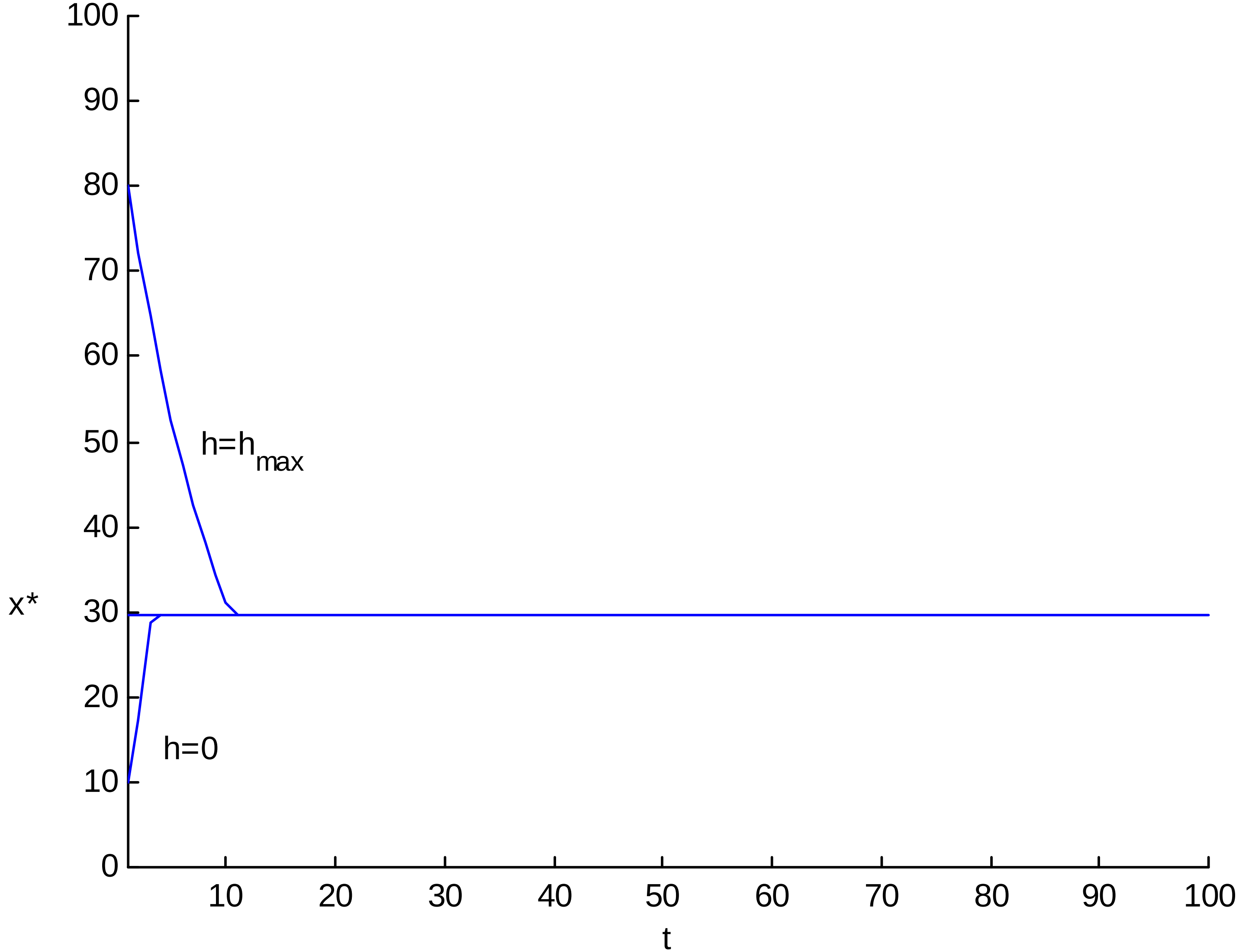
$$F'(x) - \frac{c'(x)F(x)}{p - c(x)} = r$$

- defines the optimal steady state level x*

Path to the steady state

- Optimal harvest
- h*(t) = $$\begin{cases} h_{\max}, & if\ x > x^* \\ F(x^*), & if\ x = x^* \\ 0, & if\ x < x^* \end{cases}$$

Clark-Munro –model(p=1; r=0.5; K=100; q=1; c=7; R=0.8; E_{max}=0.1; x_0 = 10 or 80, x* = 30)



Case r=0

- Same result as in Schäfer-Gordon optimum

Case r = inifinity

- Open acess case, where profits p - c(x) = 0.