Shapley-Shubik game. Solution.

Consider 5 villages around a lake. Each village takes its drinking water from the lake, and discharges its sewage into the lake. Each faces the choice to treat the sewage before discharging it or not. The costs of treating the sewage before discharging is 50.000 per year. On the other hand the yearly costs for cleaning the water it takes from the lake to make it drinkable is 20.000 times the number villages that *do not treat* their sewage before discharging it.

Task1. Fill in the table that shows annual total cost to a village as a function whether it treats or does not treat the sewage and of the number of other villages that treat their sewage.

Let us denote

- N=5 total number of villages,
- k – number of villages other than *i* that treat
- total costs (TC) of each village depend on whether it treats or does not treat

  TC(treat) = 50 + 20 (5 – (k+1)),

  TC(not treat) = 20 (5 - k).

k	0	1	2	3	4
treat	130	110	90	70	50
not treat	100	80	60	40	20

Task 2. show that the game belongs to Prisoner's dilemma types of coordination games.

It can be seen from the table that 'Not Treat' is dominant strategy (individual total costs are lower). Not treat is unique Nash equilibrium of the game, however situation when all villages treat water, is preferable for all villages.

Task 3. What is the smallest size of coalition of villages which could benefit by having its members choose Treat? (Assume that the farmers not in the coalition would continue to pursue their individual self-interest by choosing Not Treat)

The smallest size of the coalition is 3: TC(treat)=90 (coalition of 3 players who jointly treat) versus TC(not treat)=100.

Task4. If such a coalition is formed, would it better to be in coalition or free-rider.

It is better not to be in the coalition (remember that Not treat is dominant strategy). So even though a coalition of three will save on the sewage water treatment costs, such a coalition is not easily formed, as everyone prefers that others join the coalition.