# Coalitions and Stability

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# Introduction

## Prisoner's Dilemma

| | | Country Y | |
|---|---|---|---|
| | | Pollute | Abate |
| Country X | Pollute | 0,0 | 2, -1 |
| | Abate | -1,2 | 1,1 |

How to transform the game to make (*Abate*, *Abate*) a stable solution?

- binding agreement
- penalties
- altruims
- side payments

# Prisoner's Dilemma transformed

Fine of '-2' units for playing unilaterally 'Pollute'

Country Y

| Country X | | Pollute | Abate |
|---|---|---|---|
| | Pollute | 0,0 | 0, 1 |
| | Abate | 1,0 | 1,1 |

How does the outcome of the game change? Does it deter free-riding? Is cooperation self-enforcing?

# Altruism and Side Payments effects

Game 1

| Country X | Country Y: Pollute | Country Y: Abate |
|---|---|---|
| Pollute | 0,0 | -1, -1 |
| Abate | -1, -1 | 1,1 |

Game 2

| Country X | Country Y: Pollute | Country Y: Abate |
|---|---|---|
| Pollute | 0,0 | 2, -1 |
| Abate | -1, 2 | 3 ,1 |

# N-person games. Example

Let $N = 10$ identical countries

Abating effort cost $C = 7$ and confer benefits $B = 7$

# N-person games

Let $N \geq 2$
Barrett 1997

Let $K$ be number of cooperators who contribute to public good. Then players' payoffs are

$$\pi_p = a + bK, \text{ if country pollutes}$$

$$\pi_a = c + dK, \text{ if abates}$$

Cooperation size is determined by $K$

The structure of the payoff is critical in determining whether cooperation can be sustained

# N-person games. Example

$$a = 0, b = 5, c = -7, d = 5$$

Number of abating countries other than $i$

| Country i | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Pollute | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| | Abate | -2 | 3 | 8 | 13 | 18 | 23 | 28 | 33 | 38 | 43 |

- Prisoner's dilemma revisited
- What is non-cooperative solution?
- Is cooperative solution stable?

# N-person games. Example

Find non-cooperative and cooperative solutions to the game if

$$a = 12, b = 3, c = -7, d = 7$$

What is the minimum participation requirement to induce cooperation?

| | | Number of abating countries other than *i* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Country i | Pollute | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 |
| | Abate | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |

# N-person games. Example

Find solution to the game if

$$a = 0, b = 5, c = 3, d = 3$$

What is the size of cooperation?

| | | Number of abating countries other than $i$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Country i | Pollute | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| | Abate | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |

# Basic Game-theoretic conclusions

Two main reason for using game theory

- Positive: Explain some observes real-world behavior
- Normative: How to reach certain desirable outcome

Summary of the simple model of the lecture

- Environmental problems can 'solve themselves' if they are privately beneficial for all the countries
- Environmental problems can be socially optimal but not privately optimal - very hard to solve
- Sometimes there are several equilibria and countries must to coordinate to pick the right ones

# IEAs and Games with continuous set of strategies

# Static games with continuous set of strategies

Previously: assumption of a simple binary choice decision.

In application to the problem of environmental cooperation it implies that a country need to decide whether 'to participate in an environmental agreement ' or 'do not participate in the agreement'. In other words, 'all or nothing'.

Even if decision is 'to participate in the agreement', the country faces a further choice to make: by how much should it agree to participate.
Now we generalize our previous discussion by allowing countries to negotiate levels of contribution.
Assume: Public good provision - reduction of pollution.

# Static games with continuous set of strategies

Previous analysis has shown three types of possible outcomes, regarding possible cooperation:

- non-cooperative
- cooperative
- some cooperate but others do not

Let us start our analysis by considering the first two options

# Static games with continuous set of strategies

Consider $N$ identical countries, indexed by $i = 1, \ldots, N$.

**Each country payoff function**.
Each country $i$ maximizes some net benefit (i.e. payoff) function $\pi_i$. Let $q_i$ denote pollution reduction by country $i$, $q_i \in [0, \bar{q}_i]$. Total amount of reduced pollution is $Q = \sum_{i=1}^{N} q_i$.

$$\pi_i = B(Q) - C(q_i), \quad \text{for } i = 1, \ldots, N.$$

Net benefit (payoff) is given as *benefit from total reduction* of pollutant $B(Q)$ minus *individual costs* of reduction $C(q_i)$.

# Public good game
## Non-cooperative behavior

Non-cooperative behavior means that each country $i$ chooses its level of $q_i$ so that

$$\max_{q_i} \pi_i$$

without regard for the consequences for other countries.
F.O.C.:

$$\frac{\partial B(Q)}{\partial Q}\frac{\partial Q}{\partial q_i} = \frac{\partial C(q_i)}{\partial q_i}.$$

Notice that $\partial Q/\partial q_i = 1$ and given symmetry, we obtain

$$\frac{\partial B(Q^{nc})}{\partial Q} = \frac{\partial C(q^{nc})}{\partial q}, \quad \text{where } Q^{nc} = \sum_{i=1}^{N} q_i^{nc}.$$

Here subscript $nc$ means 'non-cooperative'.
Conclusion: each country abates up to the point where its own marginal benefit equals to marginal costs of pollution reduction.

# Public good game
## Full cooperative behavior

Full cooperative behavior means that all $N$ countries *jointly* choose levels of $q_i$, $i = 1, \ldots$ so that maximize their collective payoff

$$\max \Pi = N \cdot B(Q) - \sum_{i=1}^{N} C(q_i).$$

All countries act as one player!

# Public good game
## Full cooperative behavior

F.O.C.:

$$N \frac{\partial B(Q)}{\partial Q} \frac{\partial Q}{\partial q_i} = \frac{\partial C(q_i)}{\partial q_i} \quad \text{for all } i.$$

Notice that $\partial Q / \partial q_i = 1$ and given symmetry, we obtain ($c$ means 'cooperative')

$$N \frac{\partial B(Q^c)}{\partial Q} = \frac{\partial C(q^c)}{\partial q}, \quad \text{where } Q^c = \sum_{i=1}^{N} q_i^c.$$

Condition for efficient provision of public good: each country marginal costs of emission reduction equals to the sum of marginal benefits over all recipients of the public good.

# Non-cooperative VS. Full cooperative behavior

- non-cooperative solution (Nash equilibrium) is individually rational
- full cooperative solution is collectively rational
- full cooperative solution is Social optimum (doing the best for the whole society)
- full cooperation requires existence of supranational organization

# Non-cooperative VS. Full cooperative behavior

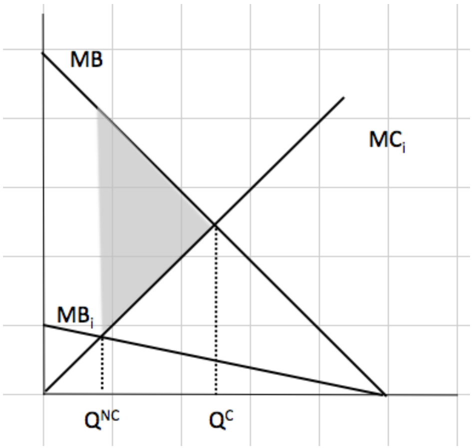



Figure: A comparison of the non-cooperative and full cooperative solutions to an environmental public good problem

# Non-cooperative VS. Full cooperative behavior

It shows

- the amount by which full cooperation abatement exceeds non-cooperative abatement ($Q^c - Q^{nc}$)
- magnitude of efficiency gain from full cooperation (the shaded triangle area in the figure)

It depends

- the relative slopes of the $MB_i$ and $MC_i$ curves
- the number of competing countries $N$ (determines the relative slopes of the $MB_i$ and $MB$ curves)

# Example. Public good game

Assume the world consists of two countries $X$ which is poor and $Y$ which is rich.
The total benefits $(B)$ and costs $(C)$ of emissions abatement $(q)$ are given by the functions

$$B_X = 8(q_X + q_Y), \qquad B_Y = 5(q_X + q_Y),$$
$$C_X = 10 - 2q_X + 0.5q_Y^2,$$
$$C_X = 10 - 2q_X + 0.5q_Y^2$$

Obtain

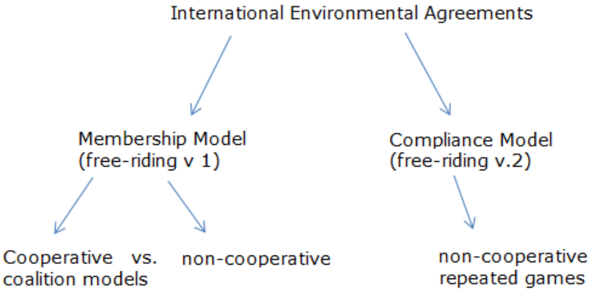
- non-cooperative equilibrium levels for both countries $X$ and $Y$ (Nash equilibrium)
- cooperative levels (social optimum)
- the payoff levels for $X$ and $Y$ in both cases
- does the cooperative solution deliver Pareto-improvements for each country? Or, would one have to give side-payments to the other to obtain Pareto improvements for each with cooperation?

## Partial cooperation

# IEA Structure

| Main Features | Sub Features | Characteristics | |
|---|---|---|---|
| Time | Framework | Implicit dyn | Explicit Dyn |
| | Horizon | | Finite or Infinite |
| | Interval | | Discrete or continuous |
| Payoff | Structural Relations | Independent (flow pollution) | Dependent (stock pollution) |
| | Arguments | only material pfs | also non-material pfs |
| | Transfers | No | Yes |
| Equilibria | Sanctions | different degree | of harshness and credibility |
| | Deviations | Single | Multiple |
| Number of issues | | Single | Many |
| Rules of | sequence of coalition formation | simultaneous | sequential |
| Coalition | number of coalition | single | multiple |
| Formation | membership | open | exclusive |
| | consensus | different degree of | consensus wrt membership |

# IEA modeling



# Membership game. Conjectural variation model

International environmental agreements (IEA) as **conjectural variation models**

d'Aspremont and Gabszewicz (1986), d'Aspremont et al. (1983), Barrett (1991,1992), Carraro and Siniscalco (1991), Barrett (1994), Bauer (1992), Hoel (1992)

Stage game (two or three stages)

# Membership game. Conjectural variation model

1st stage: players decide on participation (binary choice: participate or not)

- coalition $S$ is unique by model assumption, $|S| = K$, $K \leq N$
- 'signatories': agree to reduce pollution by negotiated amounts
- 'free-riders': act independently, doing the best they can given what the cooperators have agreed.

2nd stage: players decide on emission levels

- signatories choose their emissions cooperatively by maximizing aggregate welfare
- signatories act non-cooperatively towards free-riders
- free-riders act as singletons and choose their emission levels non-cooperatively doing the best they can given what the cooperators have agreed

3rd stage: allocation of welfare gains

- happens if players are asymmetric
- allocation rules

# Structure of Membership Game on Public Good

Assumptions in IEA modeling: in bold assumptions used further in the slides!

**1. Stage: Participation strategies**

| | | |
|---|---|---|
| Sequence | **Simultaneous** | Sequential |
| Agreements | **Single** | Multiple |
| Membership | Open | Exclusive (majority/unanimity) |
| Min. participation clause | Yes | **No** |

**2. Stage: Emission decision**

| | | |
|---|---|---|
| Emission | **efficient** | bargaining or consensus |
| Abatement | | |
| Payoffs | **static**/dynamic | tipping points |
| Parameter Values | **known** | unknown |
| Other Strategies | geoengineering, | adaptation etc. |
| Other Payoff Components | additional benefits | or effects |

Allocation of the coalition gain through *Transfers*

# Membership Model. Conjectural variation model

Sequence of moves in the first two stages

- *Nash-Cournot assumption*: players choose their moves simultaneously in both stages (Carraro and Siniscalco 1991, Bauer 1992)
- *Stackelberg assumption*: players choose their participation strategies simultaneously but emission levels sequentially (Barrett 1991, 1992)

Strictly speaking, *Stackelberg* assumption means that the second stage consists of two sub-stages

Usually, coalition (or its member, signatories) act as *Stackelberg* leader

# Membership Model

A self-enforcing agreement is such an agreement if it creates incentives for all the parties (both cooperating countries and free-riders) to adhere to the agreement once it has come into effect.

- no incentives to renegotiate
- payoffs must be such that cheating is deterred
- penalties to countries other than $i$, should not be a disincentive for a country $i$
- penalties to country $i$ should not encourage to renegotiate

# Membership Model. Conjectural variation model

Three conditions of conjectural variation models

- C1 profitability
- C2 internal stability
- C3 external stability

# Membership Model

Let $S$ be coalition of signatories of size $K \leq N$
Denote $\pi_i^S$ payoff of signatory $i \in S$ and $\pi_j^F$ payoff of free-rider $j \notin S$

## Self-enforcing agreement

Coalition $S$ is self-enforcing if

1. no signatory can gain by unilaterally withdrawing from the agreement

$$\pi_i^S \geq \pi_i^{F \cup i}, \quad \forall i \in S,$$

2. no signatory can gain by unilaterally acceding the agreement

$$\pi_j^F \geq \pi_j^{S \cup i}, \quad \forall j \notin S.$$

# Membership Model

General description of a self-enforcing agreement

### A self-enforcing international environmental agreement

is an equilibrium outcome to a negotiated environmental problem that has the following properties:

- there are $N$ countries in total, of which $K$ choose to cooperate and so $N - K$ do not cooperate
- each cooperating country selects an abatement level that maximizes the aggregate payoff of all countries that cooperate
- each free-rider country pursues its individually rational unilateral policy.

# Public good game. Membership Model

Main results of the research for **symmetric players**

- free-riders and signatories are both better off if all countries cooperate (property of Prisoner's dilemma)
- free-riders do better than signatories (property of Chicken game)
- full cooperation is not stable
- when $N$ is large, cooperation can achieve very little, no matter how many signatories there are

# Public good game. Effectiveness

Effectiveness:

- effectiveness is measured as a difference between Nash and cooperative outcome
- effectiveness of real IEAs is limited
- codifying actions that countries are already doing or would be doing without an agreement (e.g. Montreal protocol, Biodiversity Convention)

# Public good game. Enhancing cooperation

Concept of self-eforcement proves to be a useful way of thinking
On the other hand, it creates a degree of pessimism
Is there a way to achieve larger benefits from cooperation?

- role of commitment
- transfers and side payments
- linkage of benefits and costs and reciprocity
- repeated games
- etc.

# Conclusion

- if number of countries involved into environmental problem is small, then cooperative bargaining agreements are relatively easy to obtain
- if number of countries involved into environmental problem is large, then successful cooperation is harder to achieve
- difficulties are lessened if there are large nation-specific gains, and/or influential nations are willing to act as leaders

# Classroom work

A.J. de Zeeuw (2015) International Environmental Agreements, Annual Review of Resource Economics, Vol. 7, Issue 1, pp. 151-168, 2015 (either link below or attached pdf)
http://www.annualreviews.org/doi/pdf/10.1146/annurev-resource-100814-124943