

Two-player game

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Non-cooperative games

- Individual strategies for the players
- Reaction functions, best reply
- Nash equilibrium definition

- Stages games at different levels

- Repeated games, folk theorems, sustaining cooperative behaviour as equilibria

- Dynamic games

Why non-cooperative

- Classification: strategic (static), extensive (dynamic), coalition
- Important in fisheries non-cooperation (competition) vs cooperation
- Division not clear, almost all games have both non-cooperative and cooperative elements
- Typically in economics non-cooperative game theory dominates

What are non-cooperative games about

- How fisher's decisions interact with other fishers' decisions
- What is the best strategy for the fishers
- What is expected to happen in the fishery? Depends on rules of the game, number of players, biological factors
- Why fishers behave as they do?
- Assume rational choice

International fisheries negotiations

Nature of negotiations

- Countries attempt to sign and ratify agreements to maximise their own economic benefits
- Negotiations typically time-consuming
- Agreements not binding → self-enforcing or voluntary agreements

Explaining the tragedy of the commons

- Can we explain the seemingly irrational behaviour in the world's fisheries, overexploitation, overcapitalisation, bycatch...
- Non-cooperative game theory explains this behaviour
- Non-cooperative games vs open access (freedom of the seas)

Nash equilibrium

- Each player chooses the best available decision
- It is not optimal for any single player to unilaterally change his strategy
- There can be a unique equilibrium, multiple equilibria or no equilibria

A two-player non-cooperative fisheries game

- Assume there are n players (fishers, fishing firms, countries, groups of countries) harvesting a common fish resource x
- Each player maximises her own economic gains from the resource by choosing a fishing effort E_i
- This means that each player chooses her optimal e.g. number of fishing vessels taking into account how many the other players choose
- As a result this game will end up in a Nash equilibrium where all individual fishing efforts are optimal

Building objective functions of the players

- Assume a steady state:

$$h_i = qE_i x$$

$$\frac{dx}{dt} = F(x) - \sum_{i=1}^n h_i = 0$$

- By assuming logistic growth the steady state stock is then

Stock biomass depends on all fishing efforts

$$x = K \left(1 - \frac{\sum_{i=1}^n q E_i}{R} \right)$$

Objective function

- Players maximise their net revenues (revenues – costs) from the fishery
- $\max ph_i - c_i E_i$
- Here p is the price per kg, h_i is harvest of player i , c_i is unit cost of effort of player i

Two maximisation problems

- max $\rho_1 = ph_1 - cE_1$ & $\rho_2 = ph_2 - cE_2$

$$\rho_1 = pqE_1K \left(1 - \frac{q(E_1 + E_2)}{R}\right) - cE_1$$

FOC à reaction function

$$\frac{\partial p_1}{\partial E_1} = pqK - \frac{2pq^2 E_1 K + pq^2 E_2 K}{R} - c = 0$$

$$- \frac{2pq^2 E_1 K}{R} = c - pqK + \frac{pq^2 E_2 K}{R}$$

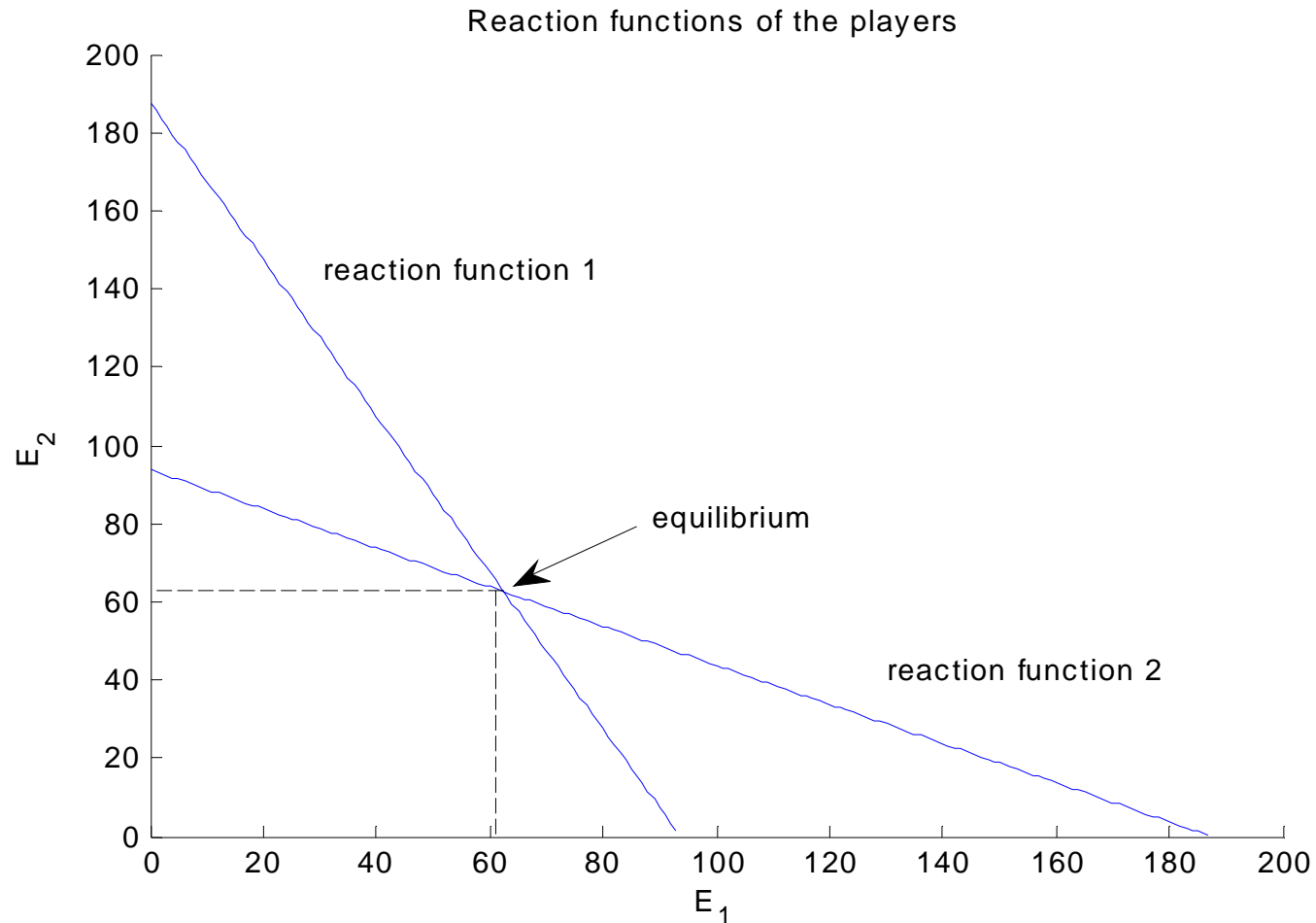
$$\Rightarrow E_1 = - \frac{cR}{2pq^2 K} + \frac{pqKR}{2pq^2 K} - \frac{pq^2 E_2 K}{2pq^2 K}$$

Country 1 reaction function

$$E_1 = \frac{R}{2q} (1 - b) - \frac{E_2}{2}$$

Nash Equilibrium

- At the intersection of the reaction functions



Equilibrium fishing efforts

$$E_1 = \frac{R}{3q}(1 - b) = E_2$$

$$E_{tot} = \frac{2R}{3q}(1 - b)$$

Equilibrium fishing efforts for n asymmetric players

- Derive by using the n reaction curves

$$E_i = \frac{nR}{(n+1)q} (1 - b_i) - \mathring{\mathbf{a}}_{j^1 i}^{n-1} \frac{R}{(n+1)q} (1 - b_j)$$

- The equilibrium fishing efforts depend on the efficiency of all players and the number of players

Illustration

- Nash-Cournot equilibrium
- Symmetric case
- Schäfer-Gordon model

Exercises

- Compute the symmetric 3-player equilibrium (effort of each player). Discuss how to compute stock, harvest and payoff based on these equilibrium efforts

A two-stage game (Ruseski JEEM 1998)

- Assume two countries with a fishing fleet of size n_1 and n_2
- In the first stage countries choose their optimal fleet licensing policy, i.e., the number of fishing vessels.
- In the second stage the fishermen compete, knowing how many fishermen to compete against
- The model is solved backwards, first solving the second stage equilibrium fishing efforts
- Second, the equilibrium fleet licensing policies are solved

Objective function of the fishermen

- The previous steady state stock is then

$$x = K \left(1 - \frac{q(E_1 + E_2)}{R} \right)$$

$$\text{where } E_1 = e_{1v} + \mathbf{a}_{w^1 v}^{n_1 - 1} e_{1w}$$

- The individual domestic fishing firm v maximises

$$\max p q e_{1v} x - c e_{1v}$$

Reaction functions

- In this model the domestic fishermen compete against domestic vessels and foreign vessels
- The reaction between the two fleets is derived from the first-order condition by applying symmetry of the vessels

$$pqK - \frac{2pq^2e_{1v}K + \overset{n_1-1}{\underset{w^1 v}{\mathring{a}}} pq^2e_w K + pq^2E_2K}{R} - c = 0$$

$$E_1 = \frac{n_1}{n_1 + 1} \left[\frac{R}{q} (1 - b) - E_2 \right] = n_1 e_{1v}$$

Equilibrium fishing efforts

- Analogously in the other country

$$E_2 = \frac{n_2}{n_2 + 1} \left[\frac{R}{q} (1 - b) - E_1 \right]$$

- By solving the system of two equations yields the equilibrium

$$E_2 = \frac{Rn_2}{q} \left(\frac{1 - b}{1 + n_1 + n_2} \right)$$

$$E_1 = \frac{Rn_1}{q} \left(\frac{1 - b}{1 + n_1 + n_2} \right)$$

Equilibrium stock

- Insert equilibrium efforts into steady state stock expression
- The stock now depends explicitly on the number of the total fishing fleet

$$x = \frac{K[1 + (n_1 + n_2)b]}{1 + n_1 + n_2}$$

Equilibrium rent

- Insert equilibrium efforts and stock into objective function to yield

$$P_1 = RpK \frac{n_1(1-b)^2}{(1+n_1+n_2)^2}$$

First stage

- The countries maximise their welfare, that is, fishing fleet rents less management costs

$$\max W_1 = P_1 - n_1 F$$

- The optimal fleet size can be calculated from the FOC (implicit reaction function)

$$\frac{\partial W_1}{\partial n_1} = RpK \frac{(1 - n_1 + n_2)(1 - b)^2}{(1 + n_1 + n_2)^3} - F = 0$$

Results

- Applying symmetry and changing variable $m = 2n+1$

$$\frac{RpK(1-b)^2}{(1+n)^3} - F = 0$$

$$n_1 = \frac{1}{2} \left[\frac{RpK(1-b)^2}{F} \right]^{1/3} - 1$$

- With $F=0$ à open access

Discussion

- Subsidies
- Quinn & Ruseski: asymmetric fishermen
- entry deterring strategies: Choose large enough fleet so that the rival fleet is not able to make profits from the fishery
- Kronbak and Lindroos ERE 2006 4 stage coalition game