Asymmetric games (bimatrix games)

In asymmetric games, the two players fill different roles (e.g., a male is interacting with a female; there is a stronger and a weaker individual, etc.). The roles are known to both players (for example, they both know which of the two is stronger). The roles may affect the pay-offs (say if the stronger never gets hurt and thus never has to pay the cost of injury in the Hawk-Dove game) or even the available strategies (if males and females have different options), but such differences may also be absent.

In an asymmetric game, we need to write down the payoffs for both roles, i.e., we have two payoff matrices ("bimatrix game"). These are often written in one table as shown below: The numbers in lower right corners of the cells are the payoffs to the individual in role A, whereas the numbers in the upper right corners are the payoffs to role B. In each case, the focal individual plays the first strategy in the parenthesis against an opponent with the second strategy. For example, $E_A(R_2,R_1)$ denotes the payoff to someone in role A if he plays R_2 against an opponent who is in role B and plays R_1.

		role B	
		R_1	R_2
role A	R_1	$E_B(R_1,R_1)$ $E_A(R_1,R_1)$	$E_B(R_2,R_1)$ $E_A(R_1,R_2)$
	R_2	$E_B(R_1,R_2)$ $E_A(R_2,R_1)$	$E_B(R_2,R_2)$ $E_A(R_2,R_2)$

In asymmetric games, strategies are conditional: "If in role A then play I, and if in role B then play J". A typical example is the Assessor strategy: If you are the stronger, play Hawk, if you are the weaker, play Dove. The shorthand notation (I,J) means "play I when in role A and play J when in role B".

ESS in an asymmetric game. A strategy pair (I,J) is an ESS if no mutant can invade a population where almost everyone is using (I,J). Assume that the mutant plays I' when in role A. It will encounter opponents in role B who play J, and its payoff is then $E_A(I',J)$. The mutant is doing less well than the resident if $E_A(I',J)< E_A(I,J)$. Similarly, a mutant J' cannot invade if $E_B(J',I)< E_B(J,I)$. If these two conditions hold for every I' and J' different from respectively I and J, then (I,J) is an ESS.

Asymmetric games have no mixed ESSs. Suppose there would be an ESS that plays a mixed strategy I when in role A. Let R_1 and R_2 be two pure strategies used by I. By the Bishop-Cannings theorem, we must have $E_A(R_1,J)= E_A(R_2,J)= E_A(I,J)$. This means that R_1 and R_2 are strategies that violate the ESS condition (e.g. with $I'=R_1$, it is not true that $E_A(I',J)< E_A(I,J)$).