Adaptive Dynamics Course
S.A.H. Geritz & É. Kisdi
Vienna 2007

The Evolution of Resource Use

Consider a population of strategies $x_1, \ldots, x_k$ and corresponding population densities $n_1, \ldots, n_k$ living off different resources with densities R_1 and R_2, respectively. The strategy x_i is the proportion of time spent searching for resource type 1, and $1 - x_i$ is the proportion of time spent searching for resource type 2. The strategy space is thus $\boldsymbol{X} = [0,1]$. The rate of offspring production per individual is some function f of the per capita acquisition rates of the two resources and their nutritional values. The per capita death rate is a constant μ. For the resource we assume a simple flow-culture dynamics. This leads to the following set of equations:

$$\begin{aligned}
\frac{dR_1}{dt} &= a_1 - b_1 R_1 - c_1 R_1 \sum_{j=1}^{k} x_j n_j \\
\frac{dR_2}{dt} &= a_2 - b_2 R_2 - c_2 R_2 \sum_{j=1}^{k} (1 - x_j) n_j \\
\frac{dn_i}{dt} &= n_i f\left(c_1 d_1 x_i R_1, c_2 d_2 (1 - x_i) R_2\right) - \mu n_i \quad (i = 1, \ldots, k)
\end{aligned} \tag{1}$$

To simplify the system, and to avoid the problem of having to establish whether there is a stable equilibrium or not, we assume that the dynamics of R_1 and R_2 are fast compared to that of $n_1, \ldots, n_k$, so that we can substitute R_1 and R_2 in the equations for $n_1, \ldots, n_k$ by their quasi-equilibrium values

$$\begin{aligned}
\hat{R}_1 &= a_1 \left(b_1 + c_1 \sum_{j=1}^{k} x_j n_j \right)^{-1} \\
\hat{R}_2 &= a_2 \left(b_2 + c_2 \sum_{j=1}^{k} (1 - x_j) n_j \right)^{-1}
\end{aligned} \tag{2}$$

The model contains still too many parameters, most of which, fortunately, can be scaled out. Let $\beta_i = a_i c_i d_i / b_i$ and $\gamma_i = c_i / b_i$ and $R_i = c_i d_i \hat{R}_i$ for $i = 1, 2$. Then (1) with (2) becomes

$$\boldsymbol{R_1 = \beta_1 \left(1 + \gamma_1 \sum_{j=1}^{k} x_j n_j\right)^{-1}}$$

$$\boldsymbol{R_2 = \beta_2 \left(1 + \gamma_2 \sum_{j=1}^{k} (1 - x_j) n_j\right)^{-1}} \tag{3}$$

$$\boldsymbol{\frac{dn_i}{dt} = n_i f\left(x_i R_1, (1 - x_i) R_2\right) - \mu n_i \quad (i = 1, \ldots, k)}$$

For the function f we take

$$\boldsymbol{f(p, q) = \left(p^{\alpha} + q^{\alpha}\right)^{\frac{1}{\alpha}}} \tag{4}$$

Different values of α correspond to the classification of resources in the following table:

Table 1.
Classification of resources

Parameter range:	**Resource type:**
$\alpha > 1$	Antagonistic
$\alpha = 1$	Perfectly substitutable
$0 < \alpha < 1$	Complementary
$-\infty < \alpha < 0$	Hemi-essential
$\alpha = -\infty$	Essential

The aim of this project is to study the evolution of the strategy x, i.e., of the partitioning of foraging time for two different resources, for the various types of resources as parameterized by α as shown in Table 1.