Adaptive Dynamics Course
S.A.H. Geritz & É. Kisdi
Vienna 2007

Predator-Prey Systems: Evolutionary Diversification of Prey

In this project, we compare the evolution of the prey species in two simple predator-prey systems. The two models are very similar, yet the results are very different.

In both cases, we consider a Lotka-Volterra type model of prey and predator, where the prey exhibits logistic population growth in absence of the predator. In Model 1, we assume the dynamics

$$\begin{aligned} \frac{dN}{dt} &= (r - hN)N - cPN \\ \frac{dP}{dt} &= ecNP - dP \end{aligned} \tag{1}$$

where N and P are respectively prey and predator densities, r is the intrinsic growth rate of the prey, h measures how sensitive the prey is to crowding, and c is the catch rate of the predator. The predator transforms the consumed prey into predator offspring with efficiency e and the death rate of the predator is d.

In Model 2, we rewrite the prey equation into

$$\begin{aligned} \frac{dN}{dt} &= r(1 - N/K)N - cPN \\ \frac{dP}{dt} &= ecNP - dP \end{aligned} \tag{2}$$

where $K = r/h$ is the prey carrying capacity. For any fixed set of parameters, the two models are obviously the same.

In both models, we assume that the prey can evolve safer strategies such that it can reduce the predator catch rate c, but this has a cost in terms of reproduction, i.e., it implies a smaller value of r. To give a concrete mechanism, c may be proportional to the fraction of time the prey is active when resting prey is well hidden from predators. With less time spent active, however, the prey can collect less food for itself and hence its intrinsic growth rate will be diminished. We thus assume that c and r are traded off such that $r = f(c)$ is an increasing function. Notice that in Model 1, the prey carrying capacity is r/h and hence is affected by c. In Model 2, however, K is a fixed model parameter.

Investigate the evolution of c under various trade-offs $r = f(c)$ including the following questions:

(i) See if any of the two models is an optimisation model, where a certain quantity is minimized or maximized and coexistence of two strategies is precluded.

(ii) What types of trade-off functions lead to evolutionary branching?

(iii) Construct an example for evolutionary branching and evolution to a dimorphic evolutionarily stable state.

It is possible to develop Model 1 further in order to consider also the evolution of the predator. Assume that the catch rate c is proportional both to the fraction of time spent active by the prey (x_1) and to the fraction of time spent active by the predator (x_2), i.e., $c = \gamma x_1 x_2$. The intrinsic growth rate of the prey increases with x_1 as above, whereas the predator death rate increases with x_2 (the predator may be hunted upon when active by a superpredator). The prey evolves x_1; the predator evolves x_2. Can the predator undergo evolutionary branching? Can you modify Model 1 such that branching of the predator becomes possible or becomes easier? (This extension is only if time permits, and the project is considered complete without the extension. It may however be interesting to contemplate this extension in the Discussion of the report even if formal analysis is not done.)