

Evolution in heterogeneous environments - II

This project is based on the Lotka-Volterra competition model discussed in the lectures. Less importantly, the population dynamics are here in discrete time. More importantly, we assume two habitat patches with dispersal inbetween, and the parameters of the Lotka-Volterra model differ between the habitats to explore the effects of habitat heterogeneity.

Consider first a habitat in isolation (no dispersal). A rare mutant with trait y grows in the equilibrium population of strategy x according to the discrete Lotka-Volterra model

$$M(t+1) = M(t) \left[1 + \rho \left(1 - \frac{\alpha(y-x)\hat{N}}{K(y)} \right) \right] \quad (1)$$

where M is the density of mutants and N is the density of residents, ρ is the intrinsic growth rate, K is the trait-dependent carrying capacity, and $\alpha(y-x)$ is the competition coefficient between strategies y and x . The resident equilibrium density \hat{N} is obtained from the resident dynamics

$$N(t+1) = N(t) \left[1 + \rho \left(1 - \frac{\alpha(0)N(t)}{K(x)} \right) \right] \quad (2)$$

Throughout we assume that $0 < \rho < 2$ such that the population exhibits a stable equilibrium. The competition coefficient is a Gaussian function of the difference in the trait values,

$$\alpha(y-x) = \exp \left[-\frac{(y-x)^2}{2\sigma_\alpha^2} \right] \quad (3)$$

We assume that the carrying capacity functions reach their maxima at two different traits in the two habitats, corresponding to different locally adapted trait values. Without loss of generality, these two trait values will be $-\theta$ and θ , respectively. The larger the value of θ is, the more the two habitats differ from one another. For the carrying capacity functions, we use the Gaussian form

$$K_1(x) = \kappa_1 \exp\left[-\frac{(x+\theta)^2}{2\sigma_K^2}\right] \quad (4)$$

$$K_2(x) = \kappa_2 \exp\left[-\frac{(x-\theta)^2}{2\sigma_K^2}\right]$$

Dispersal to the alternative habitat occurs with probability δ (which is at most 1/2). With dispersal, the mutant's dynamics are given by

$$\begin{aligned} M_1(t+1) &= (1-\delta)\tilde{M}_1(t) + \delta\tilde{M}_2(t) \\ M_2(t+1) &= (1-\delta)\tilde{M}_2(t) + \delta\tilde{M}_1(t) \end{aligned} \quad (5)$$

where \tilde{M}_1 and \tilde{M}_2 denote the densities after competition respectively in habitat 1 and 2, and are calculated as in equation (1) but with habitat-specific values of ρ and $K(y)$. Notice that the population is structured by the habitat. To calculate the invasion fitness of the mutant, we rewrite equations (5) into matrix form and substitute \tilde{M}_1 and \tilde{M}_2 explicitly:

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix}_{t+1} = \begin{bmatrix} (1-\delta)\left[1 + \rho_1\left(1 - \frac{\alpha(y-x)\hat{N}_1}{K_1(y)}\right)\right] & \delta\left[1 + \rho_2\left(1 - \frac{\alpha(y-x)\hat{N}_2}{K_2(y)}\right)\right] \\ \delta\left[1 + \rho_1\left(1 - \frac{\alpha(y-x)\hat{N}_1}{K_1(y)}\right)\right] & (1-\delta)\left[1 + \rho_2\left(1 - \frac{\alpha(y-x)\hat{N}_2}{K_2(y)}\right)\right] \end{bmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}_t \quad (6)$$

Elements of the matrix in the brackets are constant. The mutant population thus grows exponentially in the equilibrium population of the resident, and the annual growth rate of the mutant is given by the dominant eigenvalue of the matrix.

Assume first that the two habitats differ only in the position of the carrying capacity function, i.e., that $\rho_1 = \rho_2 = \rho$ and $\kappa_1 = \kappa_2 = \kappa$ (by scaling the population densities, $\kappa = 1$ can be taken without loss of generality). Find examples for different types of singular strategies illustrate them with PIPs. Construct an isocline plot and explore the coevolution of two strategies in an example with evolutionary branching.

Next, investigate how the number and stability of singularities change as a function of dispersal and as a function of θ (bifurcation analysis). Interestingly, increasing θ does not necessarily exert disruptive selection: If possible, find an example where there is evolutionary branching for small θ but no branching for large θ . Finally, extend the analysis for the asymmetric case $\rho_1 \neq \rho_2$ (concentrate of the number and stability of singularities for different levels of dispersal).