Computer practical 1

Download the Excel file pathogen_dynamics.xls. In this file, the basic SIR model (without births and deaths) is already programmed.

Explore the basic SIR model

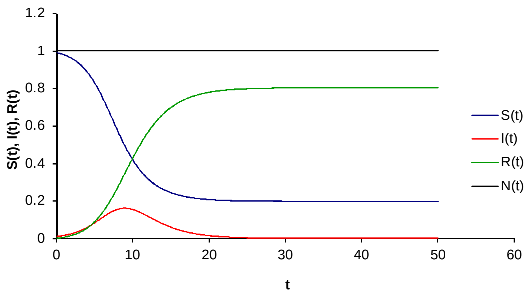
SIR model		
alpha =		
beta =	1	
gamma =	0.5	
b0 =		
mu =		
dt =	0.1	

The yellow-coloured cells at the top of the worksheet contain the transmission rate β and the recovery rate γ (the peach-coloured cells are empty and will be used later to extend the SIR model with births and deaths). The yellow cell at the bottom of the list is the length of the small time-step dt used for solving the differential equations numerically.

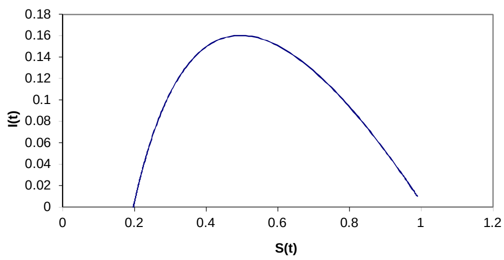
t	S(t)	I(t)	R(t)	N=S+I+R
0	0.99	0.01	0	1
0.1	0.98901	0.01049	0.0005	1
0.2	0.987973	0.011003	0.001025	1

In the main part of the worksheet, the first column (A) shows the time (t, increases with steps dt). The next three columns are the variables of the model (S, I, R) at various times. The last column shows the total population size (N=S+I+R). Note that these numbers represent units of population size (e.g. N=1 can represent 1 million individuals if the unit is a million individuals).

At time t=0, the values of S, I and R must be given (initial values). The worksheet is programmed such that at the beginning, 99% of the population is susceptible, 1% is infected and no one is recovered (check the formulas in cells B11, C11 and D11). The initial population size can be set in the yellow cell of the last column.



The first figure shows the number of susceptibles (S(t), blue), infected (I(t), red), recovered (R(t), green), and the total population size (N(t), black) against time. Without births and deaths, the total population size is constant. Explain why S(t) is only decreasing and why, after the peak of the outbreak, I(t) declines to zero (the epidemic ends).



The second figure shows I(t) against S(t), i.e., the trajectory. The epidemic starts at the rightmost point of the curve (where S is the highest) and moves along the curve until the infected disappear (I becomes zero at the leftmost point).

Exercise 1. Experiment with decreasing the total population size. Observe that

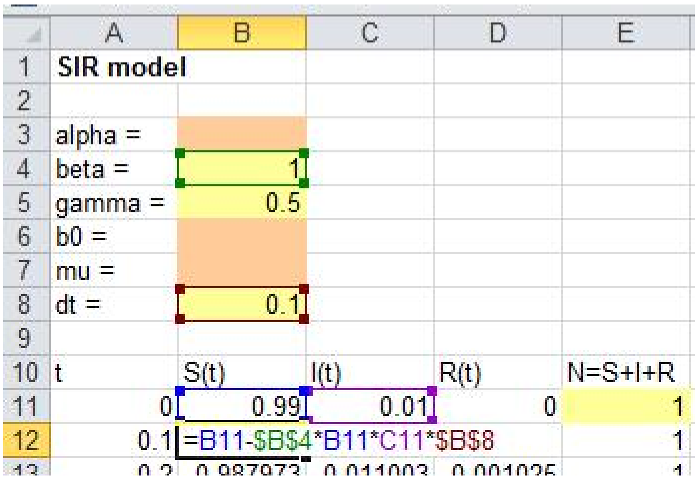
(i) the smaller the population is, the more susceptibles remain at the end

(ii) if N is less than γ / β, then there is no epidemic outbreak. Explain why this is the threshold for an outbreak.

Exercise 2. Try different values for the parameters β and γ, and record at which value of S the epidemic reaches the highest I in each case. Verify that this always occurs when S equals γ / β. Explain why I starts to decrease once S is below this threshold.

Extend the model to incorporate births and deaths

The following figure shows how the consecutive values of S(t) are calculated (the other variables are analogous). This is what you see if you click on cell B12 (the first value of S calculated from the model) and press F2:



	A	B	C	D	E
1	**SIR model**				
2					
3	alpha =				
4	beta =	1			
5	gamma =	0.5			
6	b0 =				
7	mu =				
8	dt =	0.1			
9					
10	t	S(t)	I(t)	R(t)	N=S+I+R
11	0	0.99	0.01	0	1
12	0.1	=B11-B4*B11*C11*B8			1
13	0.2	0.987973	0.011003	0.001025	1

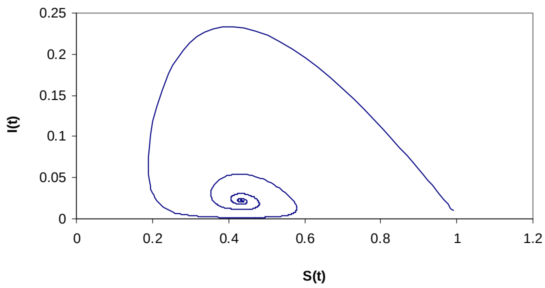
The value of S at time t=0.1 (i.e., in cell B12) is the previous value of S (in B11) plus the change. According to the model $dS/dt = -\beta SI$, which means that the change in time dt is $dS = -\beta SI \cdot dt$. This is spelled out starting with the minus sign in the formula in cell B12: cell B4 contains the value of β, B11 contains the previous value of S and C11 contains the previous value of I. **Important:** the $ sign fixes the coordinates of the cell. B4 means that it is *always* B4 what is to be used. B11 and C11 are not fixed, because when we copy the formula to the next rows, we want to use the values in the previous row (i.e., when we calculate S at time t=0.2 in B13, we want to use the values in B12 and C12). This is not the case for B4 (we don't want to use B5 etc instead), therefore it has to be fixed with the $ signs.

Exercise 3. Enter values for the disease virulence α, the population birth rate b_0, and the *per capita* death rate μ into cells B3, B6 and B7, respectively (peach-coloured). Modify the formulas in B12, C12 and D12 to include birth and death according to the equations

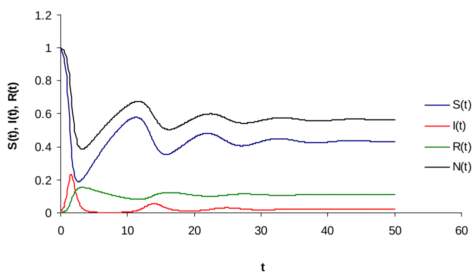
$$dS = (b_0 - \mu S - \beta SI)dt$$
$$dI = (\beta SI - (\mu + \alpha + \gamma)I)dt$$
$$dR = (\gamma I - \mu R)dt$$

You can edit the formula after clicking on the cell and pressing F2. When the cells B12, C12 and D12 are ready, copy-paste them to the rows below (the last row is 511); this will create the right formulas in the entire table (check some cells).

With the values $\beta = 6, \gamma = 0.5$ and $\alpha = 2$, $b_0 = 0.1$, $\mu = 0.1$, you should see this result:



This figure shows that after some oscillations, the population equilibrates at the innermost point of the spiral. At this equilibrium, the disease is *endemic*, i.e., there are always infected individuals present. This is because new susceptibles enter continuously via birth, so that the disease always has new hosts to infect. The S and I coordinates of the endemic equilibrium point correspond to the values S and I attain towards the right end of the other figure in the worksheet,



Notice also that N is no longer constant (it changes by births, natural deaths and disease-induced deaths).

Experiment with changing the parameter values and observe the behaviour of the model:

(i) Verify that as long as the disease is endemic, the equilibrium value of S always equals $(\mu + \alpha + \gamma)/\beta$; use the equations to explain why. Compare with exercise 2 (where μ and α are zero so that $(\mu + \alpha + \gamma)/\beta = \gamma/\beta$).

(ii) With $b_0 = 0$ and $\mu = 0$, we recover the basic SIR model (but with $\alpha + \gamma$ instead of γ). Increase b_0 and μ gradually from zero and investigate how S and I converge to the endemic equilibrium.