A First Course in Machine Vision

IMAGE ENHANCEMENT IN SPATIAL DOMAIN

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Definitions

- The principal objective of enhancement is to process an image so that the result is more suitable for a special process.
- Image Enhancement Fall into two categories: Enhancement in spatial domain and Frequency domain.
- The term spatial domain refers to the Image Plane itself which is DIRECT manipulation of pixels.
- Frequency domain processing techniques are based on modifying the Fourier transform of an image.
The term “SPATIAL Domain”

- Spatial Domain = Aggregate of pixels composing an image.
- Spatial Domain Methods = Procedures that operate directly on these pixels.
- Denoted by: \( g(x,y) = T[f(x,y)] \)
- \( F(x,y) \): Input Image, \( T \): Operator on Image
- \( g(x,y) \): Processed Image.
- \( T \) also can operate on a set of Images.
Definition of Neighborhood:

- Input for Process: A neighborhood about a point \((x,y)\)
  - The simplest form of input is a one pixel neighborhood. \(s = T(r)\)
    - \(T\): Transformation Function
    - \(s, r\): gray level of \(f(x,y)\) and \(g(x,y)\) respectively.
  - The most basic approach is rectangular sub image area centered at \((x,y)\)
  - Is it the only solution ??
Linear Transformation

- Linear Transformations:
- \( S = a \cdot r + b \)
Linear Transformation

\[ s = 1.2 \times r \]

\[ s = 0.5 \times r \]
Negative

\[ s = 255 - r \]
Other forms
Non-linear Transformation

\[ s = 255 \times \sin(r \times \pi / 255) \]
Creating a Transformation In Matlab/Octave

- Step 1: Defining the Mapping Table
- Step 2: Use the Mapping table to transfer pixel values
- Step 3: Visualization
Piece-wise linear transformation functions

- The form of them could be arbitrarily complex
- A practical implementation of some important transformations can be formulated only as piece-wise functions.
Contrast Stretching

- Is one of the simplest form of piece-wise linear function.
- Low contrast can result from:
  - Poor Illumination
  - Lack of dynamic range in the imaging sensor
  - Wrong setting of lens aperture.
Contrast Stretching example
Contrast Stretching example
Linear Transformation cont.

- Gray Level Slicing
  - Highlighting a specific range of gray levels in an image.
  - Enhancing features like masses of water in satellite imageries.
  - One Approach: show high values for pixel of interest.
  - Second Approach: Brighten the color of interest.
Slicing Example
Histogram Processing

- Histogram is a unnormalized PDF for gray values.
- It means that it contains the number of pixels for each gray value.
- So it means there are 3 Histogram for a color Image, because we have 3 Different Class of 0-255 colors.

\[ h(r_k) = n_k \]
Histogram
Histogram
Histogram Equalization

- Histograms could be considered as Probability Density Functions. So cumulative density functions could be defined as well.
Again the case of low contrast image
Hist. Equalization Result

Original Image

Enhanced Image by Histogram Equalization
Histograms after Hist. equalization
CDF’s after Hist. Equalization
Histogram Matching

- The same concept Except we use the CDF of one image as a non-linear Transform for another image.
- The resulting Image will have more or less the same Contrast as the reference.
Enhancement Using Arithmetic/Logic Operations

- Performed on a Pixel-by-Pixel basis
- Between two or MORE images.
- Example: subtraction of two images.
- Logic operations = Logic and/or nor xor xnor
- Logic operations operate on a pixel-by-pixel basis.
- We only concern about 1-“and” 2-“or” 3-“not” because they are functionally complete
- The other operations could be expresses as a function of them.
Image Subtraction

- Math formulation: \[ g(x, y) = f(x, y) - h(x, y), \]
Image Averaging

- Notation

- Application:
  - Noise Reduction
    - Astronomy
    - Surveillance
  - Background estimation

\[ \bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y) \]
Example: background estimation by Image Averaging
Spatial Filtering

- Input: Neighborhood + Sub-Image
- Output: A single value regarding to each pixel
- Definition: A neighborhood operation works with the values of the image pixels in the neighborhood and values of a sub-image that has the same dimension....

- Sub-Image is called: Mask, Kernel, Template or Window
Spatial Filters

Original Image ($f$)

Response to Mask

$$R = w(-1, -1)f(x - 1, y - 1) + w(-1, 0)f(x - 1, y) + \cdots + w(0, 0)f(x, y) + \cdots + w(1, 0)f(x + 1, y) + w(1, 1)f(x + 1, y + 1),$$
Smoothing Filters

- N-by-N filters used for noise reduction
- Blur an image and remove the high frequency part of signal
- Usually Gaussian Noise could efficiently remove by them.
- Equivalent to High-pass filters in Frequency domain.
- Most-basic type is Mask=ones(3)
Smoothing Filter Example
Order-Statistics Filters

- Median
  - Principal function: to force points with distinct gray values to be more like their neighbors.
  - Very suitable for eliminate salt and pepper noise
- Max
- Min
- Mode
- They all find an ordering-related elements in a defined neighborhood and replace it with the original pixel.
Order-statistics Filters Example

- Grab a 3x3 Neighborhood
  \{10,20,20,20,15,20,20,25,100}\}
- Sort them
  \{10,15,20,20,20,20,20,25,100}\}
- Calculate order-statistics Filter, example: Median: 20
Order-Statistics Median Example.
Sharpening

- Objective: to highlight detail in an image or enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.

- Applications are vary and important from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems.
Sharpening cont.

- Image blurring can be accomplished in the spatial domain by pixel averaging in a neighborhood.
- Averaging is analogous to integration.
- So it is logical to conclude that sharpening could be accomplished by spatial differentiation.
What will be revealed from image sharpening

- Strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at that point.
- So image differentiation will enhances:
  - Edges
  - Other discontinuities (like Noise)
- Deemphasizes:
  - Areas with slowly varying gray-level-values
Fundamental Properties of sharpening filters

- To simplify we will focus on one dimensional derivative
- We are interested on behavior in
  - Flat segments
  - At the onset and end of discontinuities (step, ramp)
  - Along gray level ramps
First order derivative properties

- Any definition for fist order should satisfy:
  - Must be zero in flat segments
  - Must be non-zero at the onset of a gray-level step or ramp
  - Must be non-zero along ramps
Second derivative properties

- Any definition should satisfy:
  - Must be zero in flat areas
  - Must be non-zero at the onset and end of a gray-level step or ramp
  - Must be zero along ramps of constant slope
Basic Definitions

- A basic definition of the first order derivative of a one-dimensional function \( f(x) \):
  \[
  \frac{df}{dx} = f(x + 1) - f(x).
  \]

- Second order derivative
  \[
  \frac{d^2f}{dx^2} = f(x + 1) + f(x - 1) - 2f(x).
  \]
1st and 2nd order Derivatives

**FIGURE 3.38**
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).
Derivatives Summary

- **1st order derivative:**
  - Produces thicker edges in an image
  - Stronger response to gray level step

- **2nd order derivative**
  - Stronger response fine details, such as thin lines and isolated points
  - Produce double response to step change in gray level
  - Their responses is greater to a point rather than a line, to a line rather than a step.

- Which one we should choose?
  - In most applications, the 2nd derivative is better suited than the 1st for image enhancement
Isotropic filters

- Their response is independent of the discontinuities in the image to which the filter applied.
- So Isotropic filters are rotation invariant, so rotating and image and then applying the filter is same as applying the filter then rotating the result.
- It can be shown (Rosenfeld and Kak [1982]) that the simplest isotropic derivative operation is Laplacian.
Laplacian

- Formula

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}. \]

- Derivative of any order is linear so Laplacian is a linear operation

- Discrete form of equation

\[
\frac{\partial^2 f}{\partial^2 x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y) \quad \frac{\partial^2 f}{\partial^2 y^2} = f(x, y + 1) + f(x, y - 1) - 2f(x, y)
\]

\[
\nabla^2 f = [f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1)] - 4f(x, y). \tag{3.7-4}
\]
### Laplacian Mask

**FIGURE 3.39**

(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).

(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

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Laplacian Mask Demo

\[
A = \begin{bmatrix}
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
-1 & 1 & 8 & -1 & -1 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0
\end{bmatrix}
\]
Sharpening by Using Laplacian

- Adding the original Image the absolute value of Laplacian operator
Simplification over the last sharpening process

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Unsharp Masking

- A process used for many years in publishing industries for image sharpening
- Consist of subtracting a blurred version of image from the image itself.
- This process called “unsharp Masking”

\[ f_s(x, y) = f(x, y) - \bar{f}(x, y) \]

- \( f_s(x, y) \): Sharpened image
- \( \bar{f}(x, y) \): Blurred Image
High-Boost Filter

- A slight further generalization of unsharp-masking is called high-boost filtering.

\[ f_{hb}(x, y) = A \cdot f(x, y) - \bar{f}(x, y) \]

\[ f_s(x, y) : \text{Sharpened image} \]

\[ \bar{f}(x, y) : \text{Blurred Image} \]

- Rewrite high-boost equation:

\[ f_{hb}(x, y) = (A - 1) \cdot f(x, y) + f(x, y) - \bar{f}(x, y) \]

\[ f_{hb}(x, y) = (A - 1) \cdot f(x, y) + f_s(x, y) \]

\[ f_s(x, y) : \text{Sharpened image} \]

\[ \bar{f}(x, y) : \text{Blurred Image} \]

- If \( A=1 \) then high-boost is a regular Laplacian.
Effect of High-Boost on a Dark Image
A First Course in Machine Vision

IMAGE ENHANCEMENT IN FREQUENCY DOMAIN
Background

- Discovered by “Jean Baptiste Joseph Fourier”.
- Published as a Theory in the book: “Analytic Theory of Heat” [1822].
- 55 years later translated in English by Freeman [1878].
- Fourier express that: any function that periodically repeats itself can be expressed as a sum of sines/or cosines of different frequencies, each multiplied by a different coefficient (we call this sum Fourier series).
Expressing a complicated periodic function by sines
One dimensional Fourier Transform

- Direct
- Inverse
- Components of Fourier Transform are complex numbers (Real and Imaginary Part)
- So we have two Images:
  - Magnitude or Spectrum of Transform
  - Phase Angle or Phase spectrum of Transform

\[
F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} \, dx
\]

\[
f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} \, du.
\]
Fourier Cont.

- Fourier
- Magnitude
- Phase Angle

\[ F(u) = |F(u)|e^{-j\phi(u)} \]
\[ |F(u)| = \left[ R^2(u) + I^2(u) \right]^{1/2} \]
\[ \phi(u) = \tan^{-1}\left[ \frac{I(u)}{R(u)} \right] \]
Application of DFT

- Filtering: Low-pass and High-pass
- Convolution in Spatial Domain is a simple multiplication in Frequency Domain
- So Mask filters are easier in Frequency under the condition that we have specialized hardware which can run DFT very fast.